Piaget's Theory of Intellectual Development

The Years 2 through 11

Piaget's Later Work

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The Years 2 through 11: Piaget's Later Work

This chapter deals with aspects of Piaget's later work (from approximately 1940 onward) on the child from about 2 to 11 years. As was shown in Chapter 1, this portion of Piaget's research and theory is voluminous and covers such matters as the child's conception of chance, space, geometry, movement, number, and other topics. Since we cannot review all the later work here, we shall focus on what we consider to be basic issues and concepts which reappear in and apply to almost all of Piaget's recent writings. We will consider (1) the revised clinical method, (2) the child's classification of objects or events, (3) the ability to place them in ordinal relations, (4) the concept of number (particularly its conservation over transformations), (5) the nature of mental imagery, (6) the development of memory and consciousness, and (7) some general characteristics of thought.

THE REVISED CLINICAL METHOD

We saw in Chapter 3 that Piaget's original clinical method was highly dependent on verbalizations. The examiner posed the questions in words, and the child was required to give the answers in the same way. The examiner's questions usually did not refer to things or events that were immediately present, and the problems did not always involve concrete objects

which the child could manipulate or even see. For example, the examiner might depict a child who had unwittingly broken some cups and might then ask the subject being questioned for a judgment concerning the child's naughtiness and the punishment to be meted out. In such a situation as this, the subject is required to do several things. He must interpret the examiner's description so as to picture the scene to himself; he must make a special effort to comprehend certain crucial aspects of the question, like the word "naughty"; and he must express his judgment in words.

After some experience with this method, Piaget came to feel that it was inadequate because it relied too heavily on language. The child might not understand everything said to him, particularly if the words did not always refer to concrete objects. Even if the child did understand, perhaps he could not adequately express in words the full extent of his knowledge. Consequently, Piaget modified his procedures, and the result is what we shall call “the revised clinical method” (sometimes called the “method of critical exploration”). The new method involves several features. First, the examiner’s questions refer to concrete objects or events which the child has before him. No longer must the child imagine these things merely on the basis of a verbal description. Second, an effort is made to let the child express his answer by manipulating the objects, and not solely express himself through language.

For example, let us suppose that the examiner wishes to know whether the child can form two distinct classes. To investigate the matter he might present the child with an array of circles and squares all mixed together in no order, and ask him to put together the ones that belong together, or sort out two distinct piles. What the child *does* with the objects—what sort of piles he makes—and not what he says about them, constitutes the primary data of the study. If after encouragement a child still cannot form a pile of circles separate from a pile of squares, then the examiner might conclude that he does not have the classification skills under investigation. While completely nonverbal tests are desirable, it is often hard to invent them. This is especially true for Piaget, since he usually investigates the child’s understanding of abstract concepts that are not easily manifested in the behavioral manipulation of concrete materials. The revised clinical method, therefore, must often depend for its data on the child’s verbal responses. But even when this is necessary, the child’s answers refer to a problem stated in terms of concrete materials which are present.

Third, Piaget introduced the use of *counterarguments* or *countersuggestions.* These involve presenting the child with a point of view that contradicts his own, and asking him what he thinks of the opposing view. The purpose of these counterarguments is to determine the stability and authenticity of the child’s thinking. Children who have mastered a concept will resist the countersuggestion; those who have not tend to be swayed by the contradictory argument.

A fourth feature of the revised clinical method is not new: the examiner’s questioning is flexible. Rather than employ a standardized list of questions, he modifies them or adds new ones as the situation demands. As before, Piaget still feels that there is no point either in asking a child a question that he does not understand or in failing to clarify an answer.

To summarize, the revised clinical method involves posing questions concerning concrete materials; allowing the child to “answer” by manipulating the materials, if this is at all possible; introducing counterarguments; and, as in the earlier clinical method, stating questions and pursuing answers in a flexible and unstandardized way. Whether or not the revised clinical procedure gives an accurate assessment of the child’s abilities is a matter for debate. In general, most psychologists (outside of Geneva) do not use this method in research, mainly on the grounds that it is not sufficiently standardized. We think that this attitude is mistaken, especially since there are very good reasons for avoiding standardization.[1] In any event, the revised clinical method is less exclusively verbal than Piaget’s earlier procedure and attempts to give an accurate assessment of the child’s thought processes which in large measure may be nonverbal.

CLASSIFICATION

Piaget has used the revised clinical method to study classification in the child. The preceding chapters have already touched on this and related matters, and it may be useful to review some of the material here. We saw that there is a primitive sort of motor classification in the sensorimotor period (0 to about 2 years) when the infant applies to objects in the environment abbreviations of familiar schemes. For example, Lucienne saw a toy parrot hanging above her crib and kicked her feet very slightly. This was an abbreviation of a scheme which she could quite easily have applied to the present situation. It seemed as if her action classified the parrot as a “thing to be swung.” Moreover, the abbreviation shows that the behavior was becoming internalized. Eventually it could be replaced by the thought: “That’s the parrot; that’s something I can swing.” But the abbreviated schemes are not yet instances of legitimate classification. One reason is that the schemes apply to individual objects over a period of time and not to a collection of objects. For example, Lucienne kicked from time to time whenever she saw parrots and thus indicated recognition. But this

recognition does not imply that she considered the parrots to belong to a class. Mature classification, on the other hand, involves the conception of a collection of things, whether they are immediately present or imagined. A second reason why it is not possible to credit Lucienne with classification has to do with *inclusion relations,* which will be expanded on shortly. Briefly, this refers to the ability to construct a hierarchical classification, such that toy parrots are a subclass of a larger, more inclusive class like toys in general.

From about 2 to 4 years the child begins to classify collections of objects in a way that is quite primitive. He uses the preconcept. Sometimes he fails to see that one individual member of a class remains the same individual despite slight perceptual changes, and sometimes he thinks that two different members of the same class are the same individual. Between 5 and 10 years, the child's classification is still faulty in several ways. There is the phenomenon of juxtaposition, the inability to see that several objects are indeed members of the same class. There is also syncretism, the tendency to group together a number of disparate events into an ill-defined and illogical whole.

As was pointed out, Piaget's investigations of the preconcept, syncretism, and juxtaposition, conducted in the 1920s and 1930s were preliminary and tentative. First, there existed methodological defects: the data were almost exclusively verbal so that Piaget's interpretation was based largely on what the child said. Second, Piaget's concepts—syncretism, juxtaposition, the preconcept—were somewhat vague and needed elaboration. In the 1950s Piaget returned to the study of classification in the child from about 2 to 12 years. These investigations make use of the revised clinical method; they also modify the notions of preconcept, syncretism, and juxtaposition and suggest new ways of conceptualizing the child's classificatory activities.

Some Properties of a Class

Before examining Piaget's research into classification, we must clearly understand what he means by a class. Suppose we have before us a number of objects all mixed together. The array contains a large red triangle, a small blue circle, a large pink circle, and a small black triangle. All the objects are discriminably different one from the other. That is, there is no difficulty in perceiving that any one object is different from any of the others. For example, the large red triangle is very obviously larger and redder than the small black triangle. Suppose, too, that we wish to place these objects into two different classes. One way of doing this is to put in one separate pile the large red triangle and the small black triangle. In the second pile would go the small blue circle and the large pink circle. If the original array contained additional triangular objects, regardless of their size or color, they would of course go in the first pile. Similarly all other circular objects would go in the second pile. The two piles each represent a class. Of course, we might classify the objects in another way. We could put in one pile the two small objects (regardless of their color or shape) and in the second pile the two large objects. There are usually many different classes that one may form from a given array of objects.

Piaget makes a number of points about the classes formed from the original array (for purposes of illustration consider just our first example, the class of triangles and the class of circles):

1. No object is a member of both classes simultaneously. For example, the large red triangle is in the class of triangles and not also in the class of circles. Thus, the classes are mutually exclusive or disjoint. This holds even if there are more than two classes formed. (For example, we might divide some animal pictures into the classes of lions, tigers, and elephants, all of which are disjoint.)

2. All members of a class share some similarity. For example, the small blue circle and the large pink circle both share the property of circularity. Circularity is the defining property, the crucial attribute, of the class; that is, we include in the class of circles any object which is circular. Another way of putting it is to say that circularity is the intension of the class. The defining property or intension of the other class is triangularity.

3. Each class may be described in terms of a list of its members. Instead of describing a class in terms of its defining property or intension (for example, the class of triangular objects), we may simply list the objects in the class (for example, large red triangle and small black triangle). Such a list is the extension of the class. Note that the list may involve concrete objects (like large, blue circles) or abstract ideas, events, actions, and so on (like the list of the parts of speech).

4. The defining property of a class determines what objects are placed in it. Another way of stating this is that intension defines extension, or the “field of application” of a concept. For example, if we know that one class is to be formed on the basis of triangularity and another on the basis of circularity, we can predict the content of the list of objects in each class.

These are some fundamental properties of classes, as Piaget defines them. (There are other crucial attributes too, like inclusion relations, which we will discuss later.) Piaget then asks whether the child classifies objects in accordance with these properties. When asked to group objects, does the child form mutually exclusive classes? Do his classes have defining properties which determine the list of objects in each class?

Piaget discovers three stages of development. The first two—both of which we may call *preoperational*—occur roughly during the years 2 to 7. The third stage—that of *concrete operations*—occurs roughly from the years 7 to 11.

Stage 1

To investigate classification, Piaget performed a number of experiments which used the revised clinical method. In one study, he tested a number of children from about 2 to 5 years of age. They were presented with flat geometric shapes of wood and of plastic. The shapes included squares, triangles, rings, and half-rings, all of which were in several colors. The shapes were mixed together and the child was told: “Put together things that are alike.” Sometimes additional instructions were given: “Put them so that they’re all the same” or “Put them here if they’re the same, and then over there if they’re different from this one but the same as each other” *(Early Growth of Logic, EGL,* p. 21).

The children displayed several methods of grouping the objects. One method is called the *small partial alignment.* With this method the child uses only *some* of the objects in the original array and puts them together in several ways apparently without any overall guiding plan. For example, one child began by putting six half-rings (semicircles) of various colors in a straight line; then she put a yellow triangle on top of a blue square; later she put a red square in between two blue triangles; then put squares and triangles in no particular order, in a straight line. There are several points to note about this performance. Sometimes similarities among objects determine the collection. For example, the subject whose performance was just described began with a line of half-rings. At other times the same child grouped things on the basis of no detectable similarity; that is, she put a yellow triangle on a blue square, or a red square between two blue triangles. In both of these cases, there is no similarity of either color or form.

It is clear that small partied alignments are not true classes for several reasons. One of them is that intension does not define extension; that is, no consistent defining property determined which geometric forms were put in various collections. The child does not operate under an overall guiding plan like a system of rules (defining properties) which organize the way in which he arranges the objects.

Other children of this age use the geometric figures to construct an interesting form or picture. One child arranged a number of circles and squares to represent a long vertical object and then proclaimed it to be the Eiffel Tower; another child placed a number of half-rings in between severed squares, all in a horizontal line, and described the result as a bridge. Piaget calls these productions *complex objects.* It is obvious that like the small partial alignments, and like some other types of collections not described here, the complex object is not a true class. Figures are not placed in the complex object because they share some defining property; rather, extension is determined solely by the requirements of the picture under construction.

In another investigation, Piaget presented children of the same age with nongeometric figures for classification—little toys which included people, houses, animals, and so on. Once again, the results showed an inability to form classes. One child put two dolls in a cradle, then two wheelbarrows together, then a horse. When the examiner asked the child for all the objects like a horse, she gave him all the animals and then a baby and two trees. This example illustrates the fact that although the young child may perceive similarities among the objects, these do not fully determine what objects go into the collection. That is, the child saw that all animals were in some respect similar and gave them to the examiner when asked for objects like the horse. If the child had stopped there, she might have formed a class which was based on the defining property of "animalness." However, she went on to throw in the baby and two trees. The similarity (intension) that she first perceived did not fully determine which objects were to be grouped together (extension). It is as if the child forgot about the initial defining property (animalness) and then switched to some other.

We may make several comments on these investigations. First, they make clear the nature of the revised clinical method. The examiner gives the child concrete objects to work with. The task instructions and questions are still verbal, of course, but they refer to real things that the child can manipulate. The child is required to say very little. Most of his responses are not verbal but behavioral. He does not have to say that all of the animals do or do not go together; rather, he can put them together or fail to do so.

Second, although the revised clinical method is an improvement over what was used before, we wonder whether the task was entirely clear to the child. The instructions (e.g., “Put together things that are alike”) seem rather vague and susceptible to many interpretations. We suspect that different methods of presenting the task to the child might produce entirely different results. Piaget considered this objection and tried an essentially nonverbal method. He began to classify the objects himself and asked the child to do the same thing. The result again was not true classification, but “complex objects,” and so on. While this method was not successful, it does not exhaust the possibilities. Other investigators have explored different procedures, with some success.[2]

Stage 2

Children from about 5 to 7 years produce collections that seem to be real classes. When presented with the situation described earlier, one child produced two large collections, one which contained all the polygons and the other the curvilinear forms. Moreover, each of these collections was subdivided further. The polygons, for instance, contained separate piles of squares, triangles, and so on, and the curvilinear forms involved separate collections of circles, half-rings, and so on. Thus, the child not only seems to form classes, but arranges them hierarchically, as in Figure 2. There are two general collections (polygons and curvilinear forms) at the top of the hierarchy, and these both branch out into several subcollections below (squares, triangles, etc.). The child’s activities may be characterized in several additional ways. (1) He places in the appropriate collection *all* of the objects which were in the initial array. The younger child did not do this; he left some objects unclassified. (2) Intension fully defines extension. That is, if the child defines a collection on the basis of the defining property of circularity, *all* circles go into that pile, and *none* is placed in any other pile. (3) At a given level of the hierarchy, similar defining properties are used to determine collections. For example, at the lower level of the hierarchy in Figure 2, all the collections are defined in terms of geometric form—squares, triangles, and so on. It is not the case that some collections are defined by form and some by color. To summarize, it would seem that the child from about 5 to 7 years produces rather elaborate hierarchical collections which deserve to be called true classes.

FIGURE 2
Classification of geometric objects.

Piaget feels, however, that the child of this stage fails to comprehend one crucial aspect of the hierarchy he has constructed. The child does not understand key relations among the different levels of the hierarchy. This is the problem of *class inclusion* which we will now illustrate. Suppose we are given a randomly organized array of blue and red squares and black and white circles. We construct an arrangement (see Figure 3) such that there are two major collections (squares versus circles) and within each of these there are two further subdivisions (blue versus red squares and black versus white circles). Thus, there is a hierarchy whose higher level is defined by shape and whose lower level is defined by color. Consider for the moment only one-half of the hierarchy, namely, the squares which are divided into blue and red. If we understand inclusion relations, then we can make statements of this sort: (1) *All* of the squares are either blue or red. (2) There are more squares than there are blue squares. (3) There are more squares than there are red squares. (4) If the red squares are taken away from the squares, then the blue ones are left. (5) If the blue squares are taken away from the squares, then the red ones are left. (6) All the blues are squares, but only some of the squares are blue. These, then, are some of the possible statements about inclusion relations—the relations of the parts to the whole, of the whole to the parts, and the parts to the parts. They may seem very obvious, but so do many other principles which children fail to understand.

FIGURE 3
Classification of squares and circles.

Piaget investigated the understanding of inclusion relations in children of various ages. Let us consider now the child from about 5 to 7 years. Piaget presented each of his subjects with a number of pictures of flowers and other things. The child was first required to group the pictures in any way he wished, and then he was asked a number of questions concerning inclusion relations. The results concerning spontaneous classification replicated what was found earlier: the child from 5 to 7 years constructs collections which seem to involve a hierarchy. One child formed two large collections: flowers versus other things; then he further subdivided the flowers into primulas versus other kinds of flowers. In terms of Figure 4, the child seemed to have constructed the top two levels of the hierarchy. (He did not make a further subdivision in terms of yellow versus other primulas.) It would seem that the construction of such a hierarchy implies the understanding of inclusion relations. If the subject divided the flowers into primulas versus other kinds, must he not understand that there are more flowers than there are primulas? The results of Piaget's questioning, however, point to different conclusions. Consider this protocol of a child aged 6 years 2 months:

> A little girl takes all the yellow primulas and makes a bunch of them, or else she makes a bunch of all the primulas. Which way does she have the bigger bunch?—*The one with the yellow primulas will be bigger*. [He then counted the yellow primulas and the other primulas and found that there were four of each kind] *Oh no, it's the same thing*. . . .—And which will be bigger: a bunch made up of the primulas or one of all the flowers?—*They're both the same.* (*EGL*, p. 102)

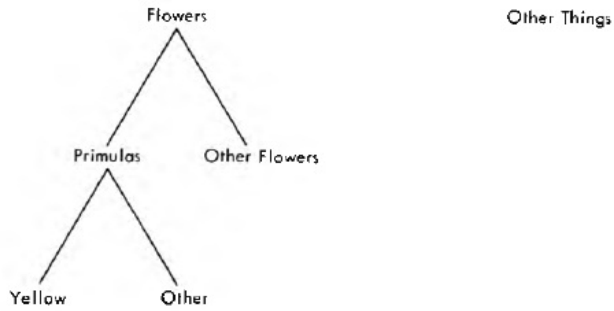

FIGURE 4
Classification of flowers and other things.

Although this child had earlier constructed a hierarchical arrangement of the materials, he maintained that the yellow primulas did not form a smaller collection than the primulas as a whole and that the primulas did not form a smaller collection than the flowers as a whole. Both of these answers, of course, are quite wrong. In both cases, the part is smaller than the whole from which it derives.

What is the explanation for the child's inability to comprehend inclusion relations? Piaget postulates that once the child has divided a whole into two subgroupings, he cannot then think simultaneously in terms of the larger collection and the subdivisions which he has constructed from it. For example, suppose a child divides a collection of flowers (the whole) into primulas versus other flowers (subdivisions of the whole). When he is asked "Are there more primulas or more flowers?" he must consider both the original collection (flowers) and one of his subdivisions (primulas) at the same time. He must compare the "size" of one against that of the other. Under these conditions, he focuses or *centers* on the collection he can see (the primulas) and ignores the original collection (all of the flowers), which is no longer present in its initial state (a collection of the primulas and other flowers all mixed together). And since he centers on the part, ignoring the whole, his answers to inclusion questions are often wrong.

Stage 3

Children from about 7 to 11 years of age are both capable of constructing hierarchical classifications and of comprehending inclusion. For example, after constructing a hierarchy, one child of 9 years and 2 months was asked:

> Which would make a bigger bunch: one of all the primulas or one of all the yellow primulas?—*All the primulas, of course, You 'd be taking the yellow ones as well.* —And all the primulas or all the flowers?—*If you take all the flowers, you take the primulas too.* (*EGL*, p. 109)

This protocol makes quite clear the child's ability to think simultaneously in terms of the whole and its parts (e.g., "If you take all the flowers, you take the primulas too"). While he physically separates the flowers into primulas and other kinds, the child is able to reason both about the original whole and its part at the same time. His thought has *decentered* from exclusive preoccupation with the part or the whole.

Piaget also found that when the child of this age was asked the same questions about hypothetical objects, the subject often failed to give correct answers. Apparently, the child's classification is *concrete:* he understands the inclusion relations of a group of real objects, but fails to comprehend the same relations when imaginary classes are involved. The gap between hypothetical and concrete reasoning is another example of *vertical décalage.*

We may summarize by stating that the child from 7 to 11 has reached the most advanced stage as far as the classification of concrete objects is concerned: he can construct a hierarchical arrangement and understand the relations among the levels of the hierarchy. Piaget then proposes that this accomplishment can be described in terms of a logicomathematical model. Let us explore this idea.

Rationale for the Use of a Logicomathematical Model

We have seen that Piaget attempts to describe the basic processes underlying the classification of objects or events. He proposes that the stage 1 child (2 to 4 or 5 years) fails to construct hierarchical arrangements partly because after a short while he forgets the defining property (intension) which he has used to form a collection. The stage 2 child (5 to 7 years) can construct a hierarchy because of the ability to use a defining property to determine which objects go in a collection, but at the same time cannot understand inclusion relations because of the inability to simultaneously consider several immediately present collections and the larger one from which they were derived. The stage 3 child (7 to

11 years) can correctly answer questions concerning inclusion because of his ability to think of original classes and their derivatives at the same time.

Thus far, we have described these basic processes (the ability to think simultaneously of subclasses and larger classes) in terms of the ordinary language. Many psychologists believe that this is the proper procedure; but others, including Piaget, feel that descriptions of structure should be phrased, as much as possible, in a formal language like mathematics.

Let us consider first, however, some aspects of the use of the common language. Most psychological theories have been stated in this way. Freud, for example, wrote exclusively in German and not in logic nor mathematics, and no doubt there is not a single formula in the entire corpus of psychoanalytic doctrine. Another example from another point on the psychological spectrum is Tolman, an experimental psychologist, who produced his theories of learning in ordinary English and made use of only a few (and nonessential) symbols. Tolman and Freud are hardly isolated examples. Today, too, the major part of psychological theorizing is done in English, or Russian, and so forth. Several advantages are usually claimed for this procedure. The ordinary language may be richer and subtler than formal languages, and also it is generally easier to read than mathematics or logic.

However, another approach to this problem is possible. Piaget feels that for scientific purposes the ordinary language is fundamentally ambiguous and must be supplemented by formal approaches. Anyone even slightly familiar with the history of psychology knows that most, if not all, psychological theories stated in the common language have been vague and easily susceptible to misinterpretation. Even today there are many fruitless arguments over the meaning of words like “concept” or “ego” or “learning.” As an example, let us consider the word “thought,” which we have used without definition quite frequently. No doubt “thought” means quite different things to different readers. To some it may mean “ideas,” and to some “consciousness”; to others it may mean “mental effort,” “meditation,” “concentration,” “opinion,” and so forth. Is it any wonder that a given psychological theory which uses words like this will elicit a variety of interpretations and, hence, considerable argument and misunderstanding? Perhaps a prime example of the difficulty is Piaget’s own use of verbal theories in his early work. Considerable confusion still surrounds the terms “egocentrism,” “moral realism,” and so forth.

Piaget feels, then, that the ordinary language produces obscure and ambiguous psychological theorizing, and must therefore be supplemented, if not replaced, by other modes of description. The physical sciences have convincingly shown that mathematics is an extremely powerful tool for communicating certain precise ideas. Piaget—along with increasingly large numbers of other psychologists—feels that it would be fruitful for psychology to adopt a similar approach, and he himself has attempted to do so in the case of classification and other matters. Let us now explore his formal description of the structure of classification.

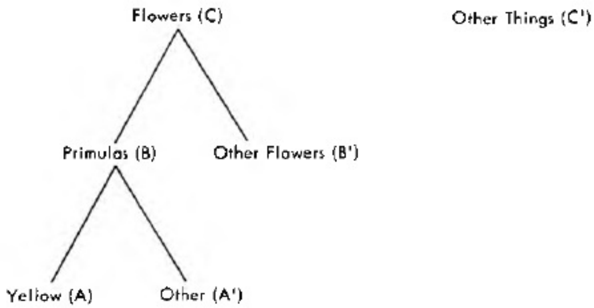

FIGURE 5
Classification hierarchy.

Grouping I

The formal description called a *Grouping*[3] begins with this situation: we have a classification hierarchy of the sort constructed by the 7- to 11-year-old children in Piaget's experiments (see Figure 5). This is what we start with (that is, it is a given) and the Grouping describes what the child can do with the hierarchy. At the top of the hierarchy that the child has constructed are the two classes, flowers which we shall symbolize as (C) and other things (C'). On the middle level of the hierarchy we find primulas (B) and other flowers (B'). On the lowest level there are yellow primulas (A) and primulas of other colors (A'). Each of the classes (A, A', B, B', C, C') is an *element* of the system. There is one *binary operator* that may be applied to the elements, namely, *combining.* We will symbolize combining by + , although the reader should be aware that combining classes is not precisely equivalent to adding numbers. The operator + is binary since it can be applied to only two elements at a time. Just as we can add only two numbers at any one time, so we can only combine two classes at a time.

Given the elements and the binary operator, the five *properties* describe the ways in which the operator may be applied to the elements.

The first property is *composition* (usually referred to in mathematics as *closure*) which states that when we combine any two elements of the system the result will be another element of the system. For example, if we combine the yellow primulas with the primulas of other colors, we get the general class of primulas. This may be written as A + A' = B. Or if we combine the yellow primulas with all the primulas, we get all the primulas. We may write this as A + B = B. This property describes aspects of the child's ability to understand a hierarchy. For example, he can mentally construct a larger class by combining its subclasses.

The second property is *associativity,* which may best be illustrated in a concrete manner. Suppose we want to combine three classes such as yellow primulas, primulas, and flowers (A, B, and C, respectively). Remember that we cannot just add all three of them together simultaneously since the operator (combining) is binary; that is, it can be applied to only two elements at a time. Given this limitation, there are at least two ways of adding A, B, and C. We might first combine the yellow primulas and the primulas and get primulas. That is, we do A + B = B. Then we might combine this result (B) with flowers-in-general (C) and get flowers-in-general. Thus, we do B + C = C. To summarize, we first perform A + B = B and then B + C = C so that our final result is C. Another way of stating this is (A + B) + C = C.

There is yet a second way of combining the classes. We could start by combining the yellow primulas (A) with the *combination* of primulas and flowers in general (B + C) and finish with the same result: flowers-in-general, (C). Thus we can write A + (B + C) = C. Note that the fined result of performing the operation by either method is C, so that the two methods may be considered equivalent. We may write this equivalence as (A + B) + C = A + (B + C). This equation expresses the fact that the child can combine classes in different orders and can realize that the results are equivalent.

The third property is *identity,* which states that there is a special element in the system (the "nothing" element), that produces no change when combined with any of the other elements. If we combine the nothing element with the yellow primulas the result will be the yellow primulas. If we symbolize nothing by 0, then we have A + 0 = A. More concretely, if we do not combine the yellow primulas with any of the other classes, then, of course, we still have the yellow primulas.

The fourth property is *negation* or *inverse,* which states that for any element (class) in the system, there is another element (the inverse) that produces the nothing element when combined with the first element. That is, if we add to the class of yellow primulas its inverse, then we are left with nothing. The inverse is equivalent to the operation of taking away the same class. If we start with yellow primulas and combine with this class its inverse, we are in effect taking away the yellow primulas with the result that we are left with nothing. We can write this as A + (-A) = 0 or A -A = 0. The inverse rule might apply to a train of thought like this: "Suppose I combine the yellow primulas with all of the other primulas. Then I have all of the primulas. But if I take away [inverse or negation] all of the other primulas, then I am left again just with the yellow primulas. ' ' Note how this train of thought is *reversible.* First, the other primulas are added, but later they are taken away, so that the thinker is once again at the point where he started. Negation, then, is one kind of reversibility.

The inverse also may be used to express aspects of class inclusion. Suppose we start with the class of primulas (B) and take away (or add the inverse of) the primulas which are not yellow (A'). This operation leaves us with the yellow primulas (A). We may write this as A = B + (-A')or A = B -A'. This type of reasoning underlies the child's ability to say that there are *more* primulas than yellow ones, that the yellow primulas are *included* in the class of primulas, or that the yellow primulas are only some of the primulas.

The fifth property actually encompasses several aspects. One of them is related to special identity elements. Suppose we combine the class of yellow primulas with itself. The result is yellow primulas. We may write this as A + A = A. In this equation, A functions as an identity element like 0. Adding A to A is like adding 0 to A: the result, A, is unchanged. Piaget calls this *tautology.* Another aspect is *resorption.* If we combine the class of yellow primulas with the class of primulas, the result is primulas. We may write this as A + B = B. Here, too, A functions as an identity element. Adding A to B is like adding 0 to B; the result, B, is unchanged. In a sense, this is another way of looking at inclusion relations. The yellow primulas must be included in the class of primulas (or must be some of the primulas) since adding the former to the latter does not change the latter.

These, then, are some of the aspects of Grouping I and are intended as a formal description of the processes underlying the child's classification. The model involves elements (classes), the binary operator of combining, and five properties governing the application of the operator to the elements.

Discussion of Grouping I

A few general remarks should be made concerning Grouping I. First, Piaget's use of mathematics is not at all meant to imply that the child understands the logicomathematical model in any explicit sense. It is obvious that most children have never heard of the special identity element, let alone Grouping I. Clearly, the child is not a mathematician at this level. In fact, he often cannot describe in *any* clear way, mathematical or otherwise, his procedure for solving a particular problem. His report is often incoherent. Piaget uses the logicomathematical model, therefore, not to characterize the child's consciousness, but to describe the processes underlying his classification.

Second, Grouping I is not metrically quantitative in the sense that it does not involve numbers. The operations involve classes which may be of any size. It does not matter whether there are 5 yellow primulas and 6 white ones, or 5,000 yellow primulas and 300 white ones. In both cases there are more primulas than there are white primulas, and so forth.

Third, we may expand on our earlier point that the Grouping is intended to describe the structure of the child's classification. Piaget is not interested in the minor details of the child's performance; that is, whether he is classifying flowers or fish or whether he first put the flowers in an arrangement and then the animals. Piaget instead attempts to capture the essence of the child's activities and to identify the processes underlying them. The Grouping is Piaget's way of describing these processes in a clear way. Therefore, the Grouping is not simply a protocol listing everything that the child does. It is instead an abstraction which describes basic processes like the ability to combine mentally two smaller classes into a larger one, or to take away one class from another.

The grouping also is a comprehensive and integrated structure. It is comprehensive since it describes the processes underlying basic classification activities. The Grouping describes the potentialities of the child, and not necessarily what he does in any one task at any one time. Let us suppose that a child constructs a hierarchy of classes. In doing so he may not make use of inclusion relations. In this case, the Grouping does not so much describe what the child actually does, but what he is capable of doing under the proper conditions.

Also, the Grouping is an integrated system in the sense that each of the properties does not stand alone but is related to all of the others. On the mathematical level, this is easy to see. The property of associativity describes the order in which elements may be combined, but the property of composition or closure is needed to interpret the result of the associative combination. In other words, associativity shows that two different orders of combining elements are equivalent, and composition reveals that both of these orders of combination result in another element which must be in the system. Thus, the property of associativity would be meaningless without the property of composition. We cannot have one property without the other. This feature of the Grouping is, of course, intended to reflect an important aspect of the child’s activities: the child’s successful classification (including the understanding of inclusion) presupposes an *interrelated whole,* a structure of mental operations. For example, suppose the child recognizes that there are more primulas than yellow primulas. This achievement implies a number of interrelated mental acts.

The child must be aware that the primulas (which are no longer present in a single collection) are the combination of yellow primulas and primulas of other colors (A + A' = B). The child must also be aware that when yellow primulas are taken away from the primulas, there remain primulas of other colors (B -A = A'). These, then, are some of the operations underlying the child’s answer to a question concerning inclusion. When the child correctly answers the question, he may not first actually perform all these operations. However, they are implicit in his answer; he could not answer correctly if it were not possible for him to perform all the operations involved in the classification system. To summarize, any particular response that the child makes to a classification problem cannot be considered in isolation. His response presupposes a complex structure, and it is this which Piaget describes as the Grouping. The Grouping, in other words, describes the mental operations which make it possible for the child to “really” understand classification.

Fourth, the Grouping explains and predicts behavior. Insofar as the Grouping describes the processes underlying the child’s classification, it may be said to explain performance. The Grouping states that the child can combine two classes to get a larger one. This operation, among others, underlies the child's ability to understand inclusion relations and in this sense explains it. Insofar as the Grouping is general it may be said to predict behavior. The Grouping is not limited to the objects Piaget used to study classification. Because the Grouping provides a description of structure, it goes beyond the details of any particular problem and allows us to predict what the child's performance is like on other similar tasks.

Fifth, Piaget has described several other Groupings all of which are intended to refer to the child's ability (from 7 to 11) to deal with concrete objects or thought about them. Therefore, stage 3 is termed *concrete operational.*

Sixth, toward the end of his life, Piaget began to feel that the Grouping model is not fully adequate as an account of the concrete operations. While the facts concerning children's performance on the classification tasks (and others as well) remain as well established as ever, the Grouping model suffers from several deficiencies. "[The Grouping] model . . . has generated little enthusiasm from logicians and mathematicians because of its unavoidable limitations . . . and consequent 'lack of elegance' " (Piaget, 1977b). (Indeed, one might even go further and claim that the logic of the model is not only inelegant, but not entirely coherent.) "[The Grouping model] . . . was too closely linked to the traditional model of extensional logic and truth tables" (Piaget, 1980, p. 5, quoted in Beilin, 1985). In view of these limitations, Piaget felt it is necessary to develop new formal models to characterize the essence of concrete operational thought. "A better way, I now believe, of capturing the natural growth of logical thinking in the child is to pursue a kind of logic of meanings" (Piaget, quoted in Beilin, 1985b). While Piaget did not have the time to develop such models in detail, he began the effort by introducing the notion of "correspondences," which we describe in our discussion of pre-operational strengths. It is important to realize, as Beilin points out, "that Piaget was not irrevocably committed to a particular logic or abstract model; consequently, following Piaget's example, others are free to [select] the logical or mathematical models that best explain the data of cognitive development" (Beilin, 1985, p. 112).

In brief, Piaget believed that while thinking is best described in terms of logical models, his own efforts in this area were not entirely successful. Hence it is necessary to expand the theory by developing new models. As Piaget claimed, he himself was the chief "revisionist" of Piagetian theory.

Summary and Conclusions

Piaget's early work (in the 1920s and 1930s) dealt with classification in a preliminary way. In the 1950s he returned to the problem, using the revised clinical method. He presented 2- to 11-year-old children with an array of objects to be classified. The findings were that in stage 1 (2 to 5 years) the child fails to use consistently a clear rule or defining property to sort the objects into different classes. He instead constructs graphic collections which are small partial alignments or interesting forms. In stage 2 (5 to 7 years), the child sorts the objects by a reasonable defining property and even constructs a hierarchical classification, but fails to comprehend inclusion relations. Stages 1 and 2 are termed *preoperational.* In stage 3, which is *concrete operational* (7 to 11 years), the child has a mature notion of class, particularly when real objects are involved. The child sorts them by defining properties, understands the relations between class and subclass, and so forth. To describe clearly the processes underlying the child's activities, Piaget proposes a logicomathematical model which he calls Grouping I. This Grouping involves some elements, a binary operator, and five properties relating the operator to the elements. Also, the Grouping is not metrically quantitative in the sense that it does not matter how big or small (in numerical terms) are the various classes involved. The child, of course is not conscious of the Grouping; rather the Grouping is intended to describe the basic structures of his activities. In his last years, Piaget recognized the shortcomings of the Grouping model and proposed the development of a new "logic of meanings."

Piaget stresses that the age norms describing classification are only approximate. A particular child may pass from stage 1 to stage 2 at 6 years and not necessarily at 4 or 5 years. One child may spend three years in stage 1 while another child may spend four years in the same stage. Piaget does maintain, however, that the *sequence* of development is invariant. The child must first be characterized by stage 1 before he can advance to stage 2 and then to stage 3. Piaget also points out that a child may not necessarily be in the same stage of development with respect to different areas of cognition. That is, a child may be in stage 1 with respect to classification, and in stage 2 of number development. Thus, a child may be slightly more advanced in some categories of thought than in others.

One important issue regarding classification, and indeed all the concepts studied by Piaget, is the generality of the findings for children in different cultures. Recently, much cross-cultural work has been carried out to determine whether children in different cultures employ the types of reasoning described by Piaget, and whether the sequence of stages is invariant across cultures, as Piaget proposes. Opper (1971; and in Dasen, 1977) has examined a number of Piagetian concepts, including classification, in rural and urban children in two Southeast Asian countries, Thailand and Malaysia. Like many other investigators (for a review, see Dasen, 1977), Opper finds that although the ages may vary, the *sequence* of development is the same in different cultures: first, Thai children are characterized by stage 1, then stage 2, and so on.

Moreover, Opper finds that Thai and Malaysian children present responses similar to those of Swiss children. For example, when a Malaysian girl in stage 2 of classification was asked whether there are more roses or flowers in a bunch of seven roses and two orchids, she responded, “There are more roses than flowers.” The examiner said, “Show me the flowers.” The child then pointed to the two orchids.

A Thai boy, in the same stage, was presented with seven roses and two lotus. He, too, maintained that there are more roses than flowers. *More roses.*—More than what?—*More than flowers.*—What are the flowers?—*Roses.* —Are there any others?— *There are.* —What?—*Lotus.* —So in this bunch, which is more, roses or flowers?—*More roses.*—Than what?— *Than lotus.*

Turning to the stage 3 child, we also find the same responses as the Swiss children. For example, a Malaysian girl said: *There are more flowers because if it’s roses, it’s only these* [pointing to roses], *but the flowers are plus these also* [pointing to orchids]. We see then that in many cases Thai and Malaysian children’s arguments are virtually identical to those of Swiss children.

How can we evaluate Piaget’s work on classification? On the one hand, Piaget has been very successful at what he has attempted to do. A number of independent investigators have confirmed that stage 1 classification takes unusual forms (e.g., Vigotsky, 1962), that young children experience genuine difficulty with class inclusion (Klahr and Wallace, 1972), and that the course of development with respect to classification is generally as Piaget has described (Kofsky, 1966). On the other hand, it should be pointed out that Piaget’s approach to classification is of a very specific sort. He focuses mainly on the hierarchical structure of classes, for example, class inclusion. He is not particularly concerned with other aspects of concepts which now seem to be quite important. Thus Neisser (1967) has pointed out that everyday concepts are often vague and difficult to define, and Rosch (1973) has developed a new approach focusing on nonlogical aspects of children's concepts. The defining property or intension of a class is often quite vague, a particular object may fit into several classes simultaneously, the boundaries between classes may be fuzzy, and it may not be possible to form a simple hierarchy. In brief, Piaget's approach focuses on only one of many important aspects of classes.

RELATIONS

In Chapters 2 and 3 we have already reviewed several aspects of relations, a problem (like classification) with which Piaget has been concerned since his earliest work in psychology. We saw that in the sensorimotor period the infant displays precursors of relations. He can broadly discriminate within the dimensions of numerosity, intensity of muscular effort, and loudness of sounds (among other dimensions). In the case of numerosity, you will recall that Laurent said "papa" when Piaget said "papa," that Laurent said "bababa" when Piaget said "papa-papa," and that Laurent said "papapapa" in response to "papapapapapapa." Laurent's imitation, although not exact, nevertheless implies an ability to discriminate or hear the difference among several sounds which differed in number of repetitions of one syllable. Similarly, in the case of muscular effort, Laurent appeared able to detect the difference among the variations in vigor with which he swung a chain, and also he was able to discriminate among sounds of different degrees of loudness. Thus, the infant can differentiate gradations within different kinds of stimuli: some things are louder than others, or more numerous, or bigger, and so forth. He can perceive differences in various aspects of his world. The ability to make such discriminations is a prerequisite for reasoning about differences.

Piaget's early research on the child from about 5 to 10 years investigated reasoning about differences, but not the perception of differences. He presented children with this verbal problem (among others): "Edith is fairer (or has fairer hair) than Suzanne; Edith is darker than Lili. Which is the darkest, Edith, Suzanne, or Lili?" *{Judgment and Reasoning,* p. 87). The results showed that children from 5 to 10 years are unable to deal with problems of this sort, called *transitivity,* at a verbal level.

As in the case of classification, Piaget returned to the problem of relations in his later work. Using the revised clinical method, he performed several interesting studies on *ordinal relations,* which we will now characterize briefly.

Some Properties of Ordinal Relations

Piaget's definition of ordinal relations involves several features. Suppose we have several numbers, such as 17, 65, 25, 3, and 1,00I. It is possible to arrange them in order of increasing size. We may use the symbol < to stand for "is a smaller number than" and write 3 < 17 < 25 < 65 < 1,00I. The sequence is an ordering of the numbers with the smallest being first, the next smallest second, and so forth. Note that the absolute size of the numbers makes no difference. The second number does not have to be exactly one more than the first or exactly twice as big as the first. The last number, so long as it is larger than 65, may be of any size whatsoever. Also, we do not need to have zero as the beginning of the series. The only requirements for ordering the numbers are that they are different from one another, that at least one number is smaller than the rest, that another is larger than all the rest, and that any number in between the smallest and the largest is both larger than the one immediately preceding it in the series and smaller than the one immediately following it. Of course, orderings are not limited to numbers. We may also order sounds on the dimension of loudness. Suppose sound *a* is very soft, *b* is much louder than *a,* and *c* is slightly more loud than *b.* Then we have $a < b < c$, where < means "is softer than." Again the precise degree of loudness does not affect the ordering.

Piaget's work deals with such matters as the child's ability to construct orderings or ordinal relations and to manipulate them in various ways. These studies, involving children from about 4 to 8 years of age, usually detect three distinct stages of development: stage 1 lasting from about 4 to 5, stage 2 from about 5 to 6, and stage 3 from about 7 and above. The first two stages are *preoperational,* and the last is *concrete operational.* While the age norms are approximate, the sequence is crucial.

Stage 1

One study was concerned with the ability to construct an ordering of a collection of ten sticks which differed only in size. We will call the shortest of the sticks (about 9 centimeters in length) *A,* the next larger *B,* and so on through *J,* the largest (about 16 centimeters in length). *A* differed from *B* by about .8

centimeters, and this also was true of *B* and C, and so on. Piaget presented the child with the sticks in a randomly organized array and asked him to select the smallest of the lot. After this was done, Piaget gave an instruction like this: "Now try to put first the smallest, then one a little bit bigger, then another a little bit bigger, and so on" (*Child's Conception of Number, CCN,* pp. 124-25). In another study the child was asked to make a staircase from the sticks.

When confronted with this problem, children in stage 1 showed severed reactions, none of which was successful. Some children produced random arrangements of the sticks, like *H, E, B, J,* and so on. Other children managed to order a few of the sticks, but not all of them. An example of this reaction is *A, B, C, D, H, F, E,* and so on.

Another strategy was to place the larger sticks in one collection and the smaller sticks in a second collection. Within each of these collections, however, the sticks were in a random order. A more advanced reaction also appeared which may be considered a transition to the next stage. The child started with some stick, like *B,* apparently selected at random; then he took another stick, like *H,* and made the top of it extend slightly above the top of *B;* a third stick, for example, *A,* was made to extend slightly beyond the top of *B;* and so forth. The result was that the tops of the sticks form an ordering; *H* is slightly higher than *B,* and *A* slightly higher than *H,* and so forth, as in Figure 6. But the bottoms of the sticks also differed in a random way, and failed to lie on a straight line as they should. Thus, the child constructs an ordering, but only by ignoring the length of each stick. This procedure frees him from the necessity of comparing each stick with the one immediately preceding it and with the one to follow. One way of characterizing these activities is to say that the child focuses (centers) on one aspect of the problem (putting the tops in order) but ignores another, equally important aspect (arranging the bottoms in a straight line). To summarize, the child at this stage frequently cannot form a systematic ordering of any number of objects although he is sometimes able to order a few of them.

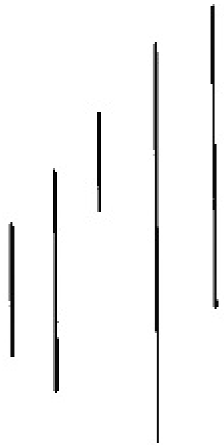

FIGURE 6
Ordering of sticks.

Stage 2

Presented with the same problem, children in the second stage generally succeed in constructing the ordinal arrangement of sticks, so that $A < B < C < D < E < F < G < H < I < J$. But the child does not build the orderings without difficulty. Sometimes he begins by ignoring the bottoms of the sticks, as in stage 1. Sometimes he makes many errors, like $A < D < B$, and so on, and takes a long time to recognize and correct them. The child continually rearranges his ordering, and shifts the sticks from one position to another. Essentially the child's procedure is one of trial and error, lacking an overall plan or guiding principle. For example, if he has chosen *A* as the smallest, he might then choose another small one, like

D, and line it up next to *A.* Then he might choose another small one, like *C,* and place it next to *D* and see that it is smaller than *D.* Since this is so, he might rearrange the sticks placing *C* after *A* but before *D.* After beginning with *A,* the child fails to look for a stick that is longer than *A* but smaller than all the ones remaining. If this rule is followed, then each step of the ordering can be constructed without any difficulty. However, the child at this stage does not employ such a logical procedure. He fails to make systematic comparisons between a given stick and the one immediately preceding it and all those following.

This tendency was further revealed by the addition of one more problem. After constructing the ordering *A* through *J,* the children were given a new collection of ten sticks, *a, b, c, d, e, f, g, h, i, j.* Each of these new sticks could fit in between a pair of sticks of the first series. That is, if the new set of sticks were ordered correctly along with the first set, the arrangement would be $A < a < B < b < C < c < D < d < E < e < F < f < G < g < H < h < I < i < J < j$. The child's task was to do precisely this; to fit the new sticks into the ordering already constructed (*A* through *J),* so as to make a new ordinal arrangement involving all twenty sticks.

Children of this stage had great difficulty with the problem. In fact, many failed to solve it. Part of one child's ordering was *C e d D,* and another produced *H g G I h j c,* and so forth. Other children succeeded in producing the correct ordering, but only after considerable trial and error.

These difficulties seem due to several factors. One factor appears to be that the child perceives the original series as a whole and finds it hard to break up the series into smaller units. Also, children of this stage do not approach the problem with a guiding principle. They fail to use a rule like, "Start with the smallest of *a-j)* insert it in between the pair of the smallest sticks in *A-J)* then take the smallest of *b-j* and insert it between the smallest pair of sticks in *B-J)* and so forth." Not only did the children fail to use a rule like this, but they also had difficulty in deciding that a given element of *a-j* was at the same time bigger than one stick in *A-J* and smaller than the next larger stick in *A-J.* To place *d* properly, the child must see that $d < E$ and that $D < d$. He must *coordinate* these two relations but fails to do so consistently. That is, some children would take *e* and, seeing that it was larger than *B,* would place it right after *B.* They failed to consider whether *e* was at the same time smaller than *C,* and therefore made an error.

After investigating the child's ability to construct an ordering and place new elements in it, Piaget went on to study the child's ability to construct equivalences between two separate orderings (which involve equal numbers of elements). To illustrate this, let us take a class with fifteen boys and fifteen girls and order each of these groups in terms of height. We find the shortest boy, the next-to-shortest boy, and so on, and we do the same for girls. We can see that the two orderings are equivalent in some ways and different in others. Some differences are that the height of the shortest boy may be 48 inches, whereas the height of the shortest girl is 44 inches. Also, the second shortest boy may be 4 inches taller than the shortest one, whereas the second shortest girl is only 1 inch taller than the shortest girl. Despite these real differences, there are important similarities between the two orderings. The boy who is 48 inches tall and the girl who is 44 inches tall, despite their difference in height, are equivalent in terms of their position in the ordering. They are both the shortest. The same holds true, of course, for the tallest boy and girl, the next to tallest, and so forth.

Piaget then raises the issue of whether the young child can recognize the equivalences between two distinct orderings. Does he understand that two objects, while differing in height, for example, can at the same time be equivalent in terms of their relative position in an ordering? To study the matter he first presented children with ten dolls, *A-J,* which were presented in a random display and which could be arranged in order of height; and with ten sticks, *A'-J',* also randomly arranged, which could be ordered in size. The sticks were smaller than the dolls, and the differences between adjacent pairs of sticks were smaller than between pairs of dolls. The child was told that the dolls are going for a walk and that each of them must have the proper stick. The intention of the instructions, of course, was to get the child to produce an ordering of the dolls and of the sticks and to make each member of one ordering correspond to the appropriate member of the other ordering. Thus, doll *A* should have stick *A*', doll *B* should have stick *B*', and so on. Piaget calls this process the placing of orderings into one-to-one correspondence.

The results showed that children of this stage can produce a one-to-one correspondence of dolls and sticks, but only in a trial-and-error fashion. The most common procedure is to order the dolls (by trial and error) and then to order the sticks (by trial and error). Only after two separate orderings have been constructed are the elements of each put into one-to-one correspondence. That is, the child first identifies the largest doll, the next to largest doll, and completes the ordering of dolls; then he goes on to order the sticks. It is only after this is done that the child places the largest stick with the largest doll, the next to largest stick with the next to largest doll, and so forth. While this procedure works, it is somewhat cumbersome. An easier method is to begin by identifying the largest (or smallest) doll and the largest (or smallest) stick and immediately placing the two together. The second step is to choose the largest doll and stick of all those remaining and to place them together, and so forth. In any event, the child in this stage does succeed in setting the two orders into one-to-one correspondence. He seems to have established that the orderings are equivalent.

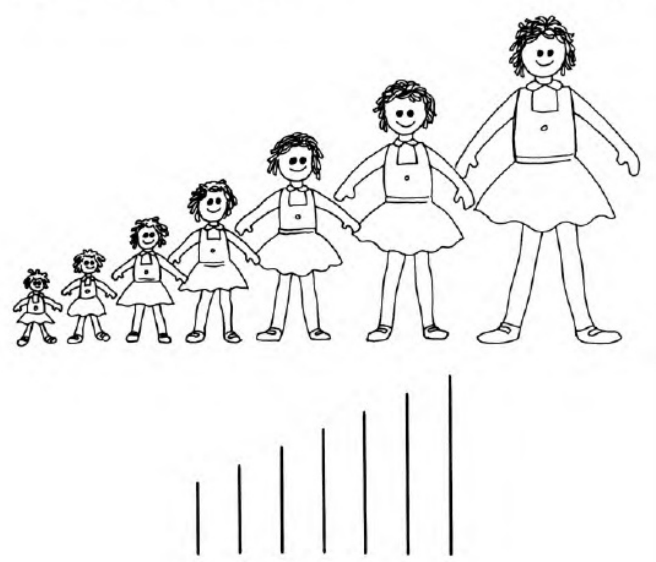

FIGURE 7
The equivalence of relative position (dolls and sticks).

The next problem concerns the stability of the equivalence established by one-to-one correspondence. Let us suppose that the sticks are placed very close together with their order preserved (as in Figure 7). The shortest stick is closest to the third tallest doll, the second stick is closest to the fourth tallest doll, and so forth. Does the child recognize that the second tallest stick is *still* equivalent to the second tallest doll, even though the former is now closest to the fourth tallest doll? That is, does the child *conserve* the equivalence of relative position when the overt one-to-one correspondence is destroyed?

Piaget presented this and similar problems to a number of children. He placed the sticks close together and asked which stick "goes with" which doll. Piaget discovered severed methods of attacking the problem. The most primitive reaction is to assert that a doll is equivalent to the stick closest to it. Thus, the second largest stick and fourth largest doll are considered to belong together simply because one is below the other. The child's judgment is dominated by *spatial* relations. Other children try to solve the problem by counting, but they fail to do so properly. For example, one child said that the fourth largest stick was equivalent to the third largest doll. The reason for his mistake was that he noticed that there were three sticks preceding the fourth largest stick; he then counted out three dolls, stopped there, and identified the third doll with the fourth stick. This method is quite frequent among children of this stage; that is, they find a doll corresponding to the nth stick, counting the preceding n -1 sticks, then count the dolls, stopping at the *n* -1th element. The child confuses the position to be found (say, stick 4) with the number of preceding elements (3).

Stage 3

After about the age of 6-7 years, the child is successful in all of the tasks we have described. When asked to construct a single ordering of sticks differing in size, the child does so quite easily. The ordering is guided by an overall plan. The child usually begins with the smallest (or sometimes, with the largest), then the next smallest, and so forth, in sequence until the ordering is complete. This strategy may be characterized as starting with the smallest and continuing to take the smallest of everything that is left, until the sticks have been exhausted. When asked to place additional sticks *(a-j)* in their proper positions within the ordering *(A-J)* already constructed, the child does so with almost no errors. The process underlying this achievement is the comparison of one of the new sticks (say, *d*) with two in the original ordering simultaneously. That is, to ascertain *d*'s proper position, the child determines that it is at the same time bigger than *D* but smaller than *E.* To phrase the matter differently, he coordinates two inverse relations—bigger and smaller than.

In a similar way the concrete operational child easily places two separate orderings into one-to-one correspondence. One child immediately put the biggest doll with the biggest ball (balls were sometimes used in place of sticks), the next to biggest doll with the next to biggest ball, and so forth. His strategy was to identify the biggest doll and ball of all those remaining and to place the two together at once. This procedure is more economical than that of the younger child who first orders the dolls, then the balls, and finally begins to put them together. When this one-to-one correspondence is destroyed, the child conserves the equivalence of relative position. He realizes that the smallest doll is still equivalent to the smallest ball and not to the ball to which it happens to be closest in space.

In summarizing the material on the concrete operational child, then, we can state that he is adept at understanding and manipulating ordinal relations. However, as in the case of classification, one limitation applies: he can deal with relations on a concrete level only; that is, when real objects or thoughts about them are involved. Nevertheless, his thought is far more advanced than that of the child in stages 1 and 2. The child can construct orderings, put two such orderings into one-to-one correspondence, and conserve the resulting equivalences. As in the case of classification, the processes underlying the child's ability to manipulate relations form integrated and comprehensive structures. Each of his mental operations cannot be understood without reference to the others of which he is capable. These processes must be interpreted in terms of complex *systems* of operations. To describe these systems, Piaget has developed several logicomathematical models, similar to Grouping I (although they, of course, dead with relations, not classes). Also, Piaget has investigated several other aspects of ordinal relations, such as *transitivity* (if $a > b$ and $b > c$, then $a > c$), which we will not cover here.

NUMBER

The ability to understand classes and relations, according to Piaget, is basic to mature concepts in many areas. The several groupings which describe the processes underlying the older child's performance in problems of classes and relations may also be used to characterize concepts of space, chance, geometry, and so forth. Since we cannot review all these concepts, we will concentrate on one that is particularly interesting and that has received considerable attention in the American and British research literature, namely, the concept of (whole) number.

First, we must understand what Piaget does and does not mean by the concept of number. He does *not* mean and is *not* interested in computational abilities as taught in the first few grades of school. Whether the child can add 2 and 2, or subtract 3 from 5, is not the issue. The reason for Piaget's lack of interest in these matters is that simple addition and subtraction of whole numbers, as well as other manipulations of them, can be carried out entirely by rote and without understanding. The child can simply memorize the addition and subtraction tables and fail to comprehend the basic concepts underlying them. Piaget does not deny that it is useful to memorize the facts of addition and subtraction; for purposes of computation, we all find it helpful to do so. He asserts, however, that for mature understanding of number, such rote memorization is not sufficient and must be accompanied by the mastery of certain basic ideas.

Among these ideas are one-to-one correspondence and conservation. Let us first consider one-to-one correspondence. Suppose we are presented with a collection or set of discrete objects as in Figure 8. The size of the objects, their color, and so forth are completely irrelevant. All that is required is that the set contain a finite number of discrete objects. We are then given a box of objects and are required to construct from it another set which has the same number property as the first set. It does not matter whether the objects in the second set (which we will call set B) are the same color, size, and so on as those in the first set (set A). Whether set A contains elephants and set B contains geraniums is irrelevant. The only requirement is that they have the same number. One way of constructing a set B so that it will have the same number property as A is by counting the objects in A (say, there are five) and then take out of the box the same number of objects. This procedure, which of course is quite adequate, probably occurs first to adults. But suppose we cannot count. Suppose we do not know the number of objects in set A. Even with these limitations there is a simple way of constructing a new set, B, which will have the same number property as A. This method merely involves putting next to each member of set A one, and only one, new object. These new objects, after the one-to-one correspondence has been established, form a set, B, with the same number as A. Of course we do not really have to physically place each new object next to one in A; we can note the one-to-one correspondence mentally. That is, we can “say to ourselves,” “This new object corresponds to the first in the line of set A,” and so on. The important idea is not the physical placing together of the sets, but the pairing of one member in set A with one in set B, however this is done.

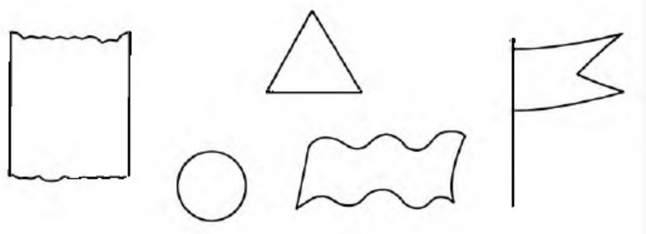

FIGURE 8
Collection of objects.

Although very simple, the idea of one-to-one correspondence is basic and powerful, and may be used in a variety of situations. If we want to determine whether there are the same number of seats as people in an auditorium, all we have to do is ask everyone to sit down (with no one allowed to sit on anyone else’s lap!). If all the people are in seats (in one-to-one correspondence with the seats) and if none of the seats is empty, then the numbers (whatever they may be) of people and seats are equal. If there are people standing, then this defines the relation of more people than seats. If there are empty seats, then this defines the relation of more seats than people. In brief, one-to-one correspondence establishes that any two sets—regardless of the nature of the objects comprising them—are equivalent in number. Counting or other procedures are not needed. Lack of one-to-one correspondence establishes that one set is larger than the other (and one smaller than the other).

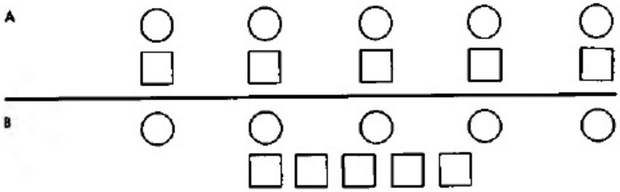

FIGURE 9
Conservation of number.

The second basic idea which Piaget investigates is *conservation.* Suppose that we have established that sets A and B are equal in number, as in Figure 9A. That is, we have put set A in a line, and below each member of set A we have put a new object. The line of the new objects is set B. Suppose that we then compress the members of set B, as in Figure 9B, so that the perceptual one-to-one correspondence is destroyed. Now each member of set B is not directly below a different member of set A. The problem is whether the two sets which now differ in physical arrangement still are equal in number. In other words, is the equivalence established in Figure 9A conserved when the rearrangement shown in Figure 9B is performed? To adults, this may seem like a foolish question. Of course, the equality of numbers has not changed! But the problem is whether children accept this simple and basic idea, too. If they do not, then their world of number must be very chaotic indeed. If quantity is seen to change whenever mere physical arrangement is altered, then the child fails to appreciate certain basic constancies or invariants in the environment.

Piaget has conducted a number of investigations on the child’s understanding of these two basic ideas: one-to-one correspondence and conservation of the equivalence of two numbers. He finds that young children fail to understand these two notions and that a period of development is required before the child achieves the mental operations necessary for thorough comprehension of number. Let us now review the experiments.

Stage 1

To study the ability to construct sets of equivalent number, Piaget presented children with a variety of problems. The simplest of these involved placing before the child a row of six or seven pennies or buttons or sweets, and so on. The examiner then asked the child to pick out the "same number" or "as many" from a large collection of similar objects. Thus the child was given set A and was required to construct a second set, B, which was equivalent in number. The children were, of course, not told how to construct set B. Here is a protocol describing how a stage 1 child, 4 years and 7 months of age, dealt with the problem. Piaget had placed six sweets in a row and told the child that they belonged to his friend Roger:

> "Put as many sweets here as there are there. Those . . . are for Roger. You are to take as many as he has. " (He made a compact row of about ten, which was shorter than the model.)—"Are they the same?"—"*Not yet*" (adding some).—"And now?"—"*Yes.*"—"Why?"—"*Because they're like that*" (indicating the length). (*CCN*, p. 75)

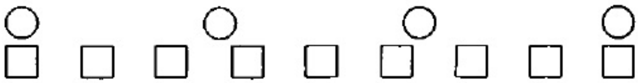

FIGURE 10
Failure to construct equal sets.

The example makes clear the predominant tendency of this stage. The child does not use the method of one-to-one correspondence. Instead, he thinks that the two sets are equivalent in number if they have the same lengths. In Piaget's terms, the child *centers* on one dimension—the length—of set A (Roger's sweets or the model) and bases his construction of set B solely in terms of that one dimension. The result is pictured in Figure 10. The lengths of the two rows are equal, but their numbers are not. The new row is denser; that is, there are smaller spaces between the sweets, than Roger's row, but the child ignores this fact and concentrates only on the lengths. Since he fails to *coordinate* the two dimensions of length and density at the same time, he cannot construct sets equivalent in number except when very small numbers are involved, or except by accident.

In another investigation, Piaget tried to make the child understand the principle of one-to-one correspondence, and then performed the conservation experiment. In this study, set A was a row of ten vases and set B consisted of flowers. One child, 4 years and 4 months of age,

> put 13 flowers close together in a row opposite 10 vases rather more spaced out, although he had counted the vases from 1 to 10. Since the rows were the same length, he thought that the flowers and vases were "*the same.* "—"Then you can put the flowers into the vases?"—"*Yes.* "—He did so, and found he had 3 flowers [left] over. (*CCN*, p. 50)

The child, then, initially constructed set B so as to make it the same length as set A and thought that the two sets were therefore equal in number. The examiner then made the child construct a one-to-one correspondence between the flowers and vases; that is, the child put each flower in a vase. The result was ten flowers in ten vases (or two sets equivalent in number), and the three extra flowers were discarded. The question now is whether the child realizes that the two sets are really equivalent in number. Does the child *conserve* the equivalence despite a mere physical rearrangement of the objects? To find out, Piaget continued the experiment with the same child.

> The flowers were taken out and bunched together in front of the vases. [That is, they formed a shorter row than did the vases.] "Is there the same number of vases and flowers?"—"*No.*"—"Where are there more?"—"*There are more vases.*"—"If we put the flowers back into the vases, will there be one flower in each vase?"—"*Yes.*"—"Why?"—"*Because there are enough.* " (The vases were closed up and the flowers spaced out.) —"And now?"—"*There are more flowers.*" (*CCN*, p. 50)

Note that after the child had himself established a one-to-one correspondence between the flowers and vases, he failed to conserve the numerical equivalence of the two sets. When the flowers were put into a shorter row than the vases, the child believed that the numbers were no longer equal and that now there were more vases. He maintained this even though he realized that the one-to-one correspondence could be reestablished; that is, that the flowers could be returned to the vases. Then when the row of vases was made shorter than that of the flowers, he changed his mind once again. He asserted that now there were more flowers. Clearly, this child centered on the lengths of the rows and used only this information to make judgments of equivalence or lack of equivalence of number. When the rows were the same length (as when the flowers were in the vases), he said that they were equal in number. When the rows differed in length, he believed that the longer line had the greater number.

Piaget also investigated the role of counting, questioning the way in which counting the two sets affects the child's judgment. One child, 5 years and 3 months of age, failed the conservation problem. He said that set A (six glasses) was greater than set B (six bottles) because one was longer than the other. Then the examiner said:

> "Can you count?"—"*Yes.*"—"How many glasses are there?"— "*Six.*"—"And how many bottles?"—"*Six.*"—"So there's the same number of glasses and bottles?"—"*There are more where it's bigger* [that is, longer]." (*CCN*, p. 45)

This examination shows that while the child can count, the act is meaningless in deeding with conservation. Although he can recite a string of numbers, he does not comprehend what they signify. The fact that he counted six bottles and also six glasses does not imply to him that the sets are equal in number. For him, equality of number is determined solely by equality of lengths, and counting is an extraneous and irrelevant act, which does not assure either the equivalence of sets or its conservation.[4]

Stage 2

The child of this stage easily constructs two sets equivalent in number, but fails to conserve the equivalence when the sets are rearranged. Per, a child of 5 years, 7 months,

> had no difficulty in making a row of 6 sweets corresponding to the model. [Piaget uses "model" to refer to set A, the row to be copied, and "copy" to refer to set B.] The model was then closed up: "*I've got more.* "—"Why?"— "*Because it's a longer line.*" (The process was reversed.)—*Now there are more there, because it's a big line.*" But a moment later, Per said the opposite: "Are there more here [referring to the longer row]?"—"*No.*"—"Why not?"—"*Because it's long.*"—"And there [the shorter row]?"—"*There are more there, because there's a little bundle*" [The child meant that the shorter row was denser].—"Then are there more in a little bundle than in a big line?"— "*Yes.*" After this Per went back to using length as the criterion, made the two rows the same length again and said: "*Now they're both the same.*" (CCN, p. 79)

The protocol shows that the child of this stage easily constructs a set equal in number to another. He also establishes the equivalence by the method of one-to-one correspondence. That is, in order to construct set B, he places a new sweet just below each in set A. But the one-to-one correspondence is not fully understood; it is just "perceptual." When set B is made shorter than set A, the child fails to conserve the equivalence which he so easily constructed. The protocol also shows that the child is ambivalent about the criteria used to establish equality or inequality of number. Sometimes he maintains that the longer row has more because it is longer; at other times he believes that the shorter row has more because it is denser. In Piaget's terms the child sometimes *centers* on the lengths (ignoring densities) and sometimes centers on the densities (ignoring lengths). This tendency is an improvement over what occurs in the previous stage, since the younger child (in stage 1) consistently centers on only one of the two dimensions, usually length, and does not consider the other, usually density, at all. By contrast, the child in stage 2 has widened the sphere of his centrations. He notices, albeit at different times, that both dimensions may be relevant and uses the information from either of these dimensions separately to make a judgment. This use of partial information is called *regulations.* We will see next how the child in the period of concrete operations *coordinates* the two dimensions.

Stage 3

The results of this stage are easy to describe. The child can now construct a set numerically equivalent to another set and can conserve their equivalence despite changes in physical arrangement. Here is a protocol illustrating this stage:

> "Take the same number of pennies as there are there [there were 6 in set A], He made a row of 6 under the model, but put his much closer together so that there was no spatial correspondence between the rows. Both ends of the model extended beyond those of the copy. "Have you got the same number?"—"*Yes*."—"Are you and that boy [referring to the hypothetical owner of set A] just as rich as one another?"— "*Yes*."—(The pennies of the model were then closed up and his own were spaced out.)—"And now?"— "*The same.*"—"Exactly?"—"*Yes*."—"Why are they the same?"—"*Because you've put them closer together*." (CCN, p. 82)

This protocol contains several interesting features. One feature is that in making set B equal to set A, the concrete operational child does not bother to place each element in B directly under each element in A. He does not need to rely on the perception of spatial proximity between the elements of each set. How then does he construct numerically equivalent sets? One method, of course, is simply to count the number of objects in set A, and then merely count out the same number for set B. Probably some children used this method, but Piaget concluded from his clinical examinations that other children did not use counting. They seemed to use the method of one-to-one correspondence, but in a more sophisticated way than the younger child. The concrete operational child's technique may be described as follows: to construct set B equal to set A, he puts out one penny for the first penny in set A, and so forth. It does not matter *where* he puts the members of set B. The only crucial requirement is that he match each member in set A with one and only one member in set B (a nonspatial one-to-one correspondence). The child must not forget to put out a penny for each member of set A (that is, he cannot skip any member of set A) and must not put out more than one penny for each member of set A (that is, he must not count any member of set A twice).

The process of establishing sets equal in number may be described in terms of classes and relations. As far as relations are concerned, the child uses the method of *vicariant ordering.* Suppose that set A (the model) is a line of pennies, and the child must construct a set B (the copy) from a large supply of candies. He begins by pointing at the penny on the extreme left and puts out a sweet. Then he points to the second penny from the left, puts out a sweet for it, and continues until the line of pennies has been exhausted. This process of pointing to one penny at a time, being careful to count each penny once and only once, is an ordering. It is equivalent to saying: "This penny comes first, this one second, this one third and so forth. In a way, the ordering of pennies is like arranging a series of sticks or dolls in order of height. There is a first stick, a second stick, and so forth, just as there is a first penny and a second one. Therefore, something like the ability to construct ordinal relations underlies the child's construction of sets equivalent in number.

Despite the evident similarity, the two processes—constructing ordinal relations (as in ordering the sticks) and vicariant ordering (the pennies)—are not identical. In the case of the sticks, there is one and only one shortest stick which must come first in the series, one and only one second shortest stick which must come second in the series, and so forth. In the case of the pennies, it does not matter which penny is considered first in the series, which comes second, and so on. One could start counting at the extreme left, at the extreme right, in the middle or wherever one pleased, just so long as one is careful not to omit pointing to each of the pennies and not to point to any of them more than once. The ordering of pennies is called "vicariant" for this very reason: the order in which the pennies are counted does not matter.

Other aspects of relations are involved too. When putting out one and only one sweet for each penny, the child is *coordinating* two orderings. This is similar to the problem of dolls and sticks. Just as the child can give to the shortest doll the shortest stick, to the second shortest doll the second shortest stick, and so forth, so can he place the first sweet with the first penny, the second sweet with the second penny, and so forth. Of course, the one-to-one correspondence of pennies and sweets is vicariant, whereas the one-to-one correspondence of dolls and sticks is not. In the latter instance, there is one and only one stick (the shortest) which goes with the shortest doll, and so forth. In the case of pennies and sweets, it does not matter which sweet is placed into correspondence with any penny, so long as one and only one sweet is used for each penny.

The construction of equivalent sets also involves *classification.* To the child, the pennies in set A, for instance, are in some ways all the same and in some ways different from one another. They are different in that a certain penny is counted first, another one second, and so forth. They are the same in that it does not matter which is counted first, which second, and so forth. In other words, it is only the child's act of pointing to each in turn that differentiates the pennies; otherwise, they are all equivalent. Insofar as each of the pennies is an element equivalent to all the rest, they are all members of the same class. The same is true, of course, of the sweets in set B.

Thus far we have seen how the child's ability to construct sets equivalent in number may be analyzed into a number of component skills. Underlying the child's overt performance (e.g., placing on a table seven sweets corresponding to seven pennies) are a number of *concrete operations:* vicariant ordering, one-to-one correspondence of two vicariant orderings, and classification. Some of the operations involve classes and others relations. Thus, number is a union of classes and relations. The operations are *concrete* since the child can apply them only to immediately present objects or thoughts about them. They are *operations* since they are actions which the child performs mentally and which have the added property of being reversible. This means that for each particular mental action, for instance addition, the child can perform its opposite action, in this case subtraction, which leaves him where he started. As operations, they may also be described in terms of overall structures or systems, that is, in terms of the Groupings, an example of which we have given in the case of classification.[5]

In the stage of concrete operations, the child can also conserve number. After constructing two sets equivalent in number, the child recognizes that the sets remain equivalent despite mere physical rearrangement of the sets. If the seven sweets are compressed to make a short line while the line of seven pennies remains the same, the two sets are nevertheless still equal in number. The equivalence has been conserved.

What enables the concrete operational child to conserve while the preoperational (stages 1 and 2) child fails to do so? Recall the mechanism underlying the preoperational child's failure: centration. The younger child centers on only a limited amount of the information available. When the row of sweets is compressed, he notices only that the line of pennies is now longer than the line of sweets. He ignores the fact that the line of sweets is denser (has smaller spaces between adjacent elements), and bases his judgment only on the lengths. The preoperational child knows that *empirical reversibility* is possible: he realizes that if the sweets were returned to their original positions, there would be one sweet for each penny. This knowledge does not help, however; despite it, he feels that the number of a set changes when its appearance is altered. Perceptual factors have too strong a hold on the child at this stage. They are not yet sufficiently controlled by mental actions which can compensate for misleading information.

By contrast, the concrete operational child decenters his attention. He attends to both the relevant dimensions and uses this information in several ways.

1. He notices that the line of pennies has become longer than the line of sweets and that the line of sweets has become denser than the line of pennies. Moreover, he coordinates the two dimensions. He mentally manipulates the visual data available to him. This mental activity leads him to realize that while the length of the line of pennies increases (relative to the sweets) by a certain amount, the density of the line of sweets increases by an equivalent amount. In other words, the child conceives that the pennies' increase in length is balanced by, or compensated for, by the sweets' increase in density: there is a relation of reciprocity or compensation between length and density. In effect, one increase cancels out the other with the result that the sets remain equivalent in number. This reciprocity is one form of reversibility. Since the increase in length counteracts the increase in density, the result is a return, or a reversal, to the original situation, which is equal number.

2. The concrete operational child also comes to use the operation of negation. We have already seen that when the row of sweets is compressed, the concrete operational child realizes that the sweets' increase in density is reciprocated by the pennies' increase in length, and that, as a result of these reciprocal transformations, the number of the two sets remains equivalent. The concrete operational child is also able to imagine that these changes can be annulled or negated. He reasons that the action of contracting the sweets can be negated by the inverse action of spreading them out. The one action is annulled by the other. Such annulment or negation is another form of reversibility; that is, the child mentally reverses the action of contracting the row of sweets. As a result he attributes equal numbers to the two sets. Note that the stage 3 child both reverses the act of contracting and recognizes that the final result is the original arrangement of sweets

and pennies. The stage 2 child, who is capable of empirical reversibility, recognizes that the sweets can be returned to their original position but does not focus on or appreciate the act of rearrangement. He attends to states, not transformations.

3. The concrete operational child sometimes uses an identity argument, reasoning that the numbers must be the same since the same objects are involved: nothing has been added or taken away.

The stage 3 child's thought is concrete in a special sense which Sinclair (1971), one of the most important Genevan investigators, expresses quite clearly: "Concrete operations . . . does not mean that the child can think logically only if he can at the same time manipulate objects. . . . *Concrete,* in the Piagetian sense, means that the child can think in a logically coherent manner about objects that do exist and have real properties, and about actions that are possible; he can perform the mental operations involved both when asked purely verbal questions and when manipulating objects. . . . The actual presence of objects is no intrinsic condition" (pp. 5-6).

To summarize, the stage 3 child, having entered the period of concrete operations, can construct two sets equivalent in number, and can conserve this equivalence despite changes in appearance. Underlying these achievements are a number of thought processes. The ability to construct equivalent sets requires *vicariant ordering* and *classification.* The ability to conserve, which is acquired as a result of the decentration of the child's attention, is supported by three types of operations which are sometimes explicitly expressed in the child's justification of his response: *reciprocity, negation,* and *identity.* These are aspects of *concrete* operations, which may be described by the *groupings.* The child does not always perform all of the thought processes when presented with a problem of constructing equal sets, nor does he refer to all three arguments when asked for a justification of conservation. He might only refer to one or perhaps two of them. The child is, however, *capable* of performing all the concrete operations, although he may not always do so. In fact, after a period of time the concrete operational child takes conservation for granted. He immediately recognizes that number is conserved and does not need to prove conservation to himself by means of negation or reciprocity. When asked why number is conserved, he thinks that the question is silly and that the fact of conservation is self-evident. For him, conservation has become a matter of logical necessity. This is evidence that the child has acquired an underlying structure of mental operations in which each is dependent upon the other and none is performed in isolation. The stage 3 child's thought is concrete in the special sense that he can think coherently about and deal with real objects but not hypothetical entities.

In conclusion, Piaget's work on number has been extraordinarily productive. It has stimulated volumes of research on children's number, and many of Piaget's findings have been successfully replicated, even in non-Western societies (see Dasen, 1977). As we shall find in Chapter 6, the work has also had implications for educational curricula. Like many major contributions to psychology, the work has aroused a good deal of controversy, and several alternative views have been proposed (see, for example, Gelman and Gallistel, 1978; and Ginsburg, 1982).

CONSERVATION

Thus far, we have described only the conservation of number—that is, the child's ability to recognize that the numerical equivalence between two sets remains unchanged despite alterations of physical arrangement. Piaget has also investigated severed other conservations which include continuous quantity, substance, weight, and volume. The *conservation of continuous quantity* may be defined by this situation. The child is presented with two identical beakers (A and B), each filled with equal amounts of liquid (see Figure 11), and is asked whether the two glasses contain the same amount or not the same amount to drink. After he agrees to the equivalence of quantities, the liquid is poured by either the experimenter or the child from one of the two identical beakers (say, B) into a third, dissimilarly shaped beaker (C). The column of the liquid in the third class (and the glass itself) is both shorter and wider than that in the remaining original glass (A). The child is now asked whether the two beakers (now A and C) contain equal amounts. If he asserts that they do, he is asked to explain why. The liquid in C is then returned to the original beaker B, and the child is again asked if A and B contain identical amounts. The manipulation is repeated, this time with a glass (D) which is taller and thinner than the original beakers. Finally, the liquid of either A or B is poured into a set (E) of about three or four smaller glasses and the same questions are asked of the child. If the child continuously asserts in each case that the amount that has been poured from B into the different beakers is always the same as the amount remaining in the original beaker (A), then he has conserved continuous quantity. That is, the child recognizes that merely pouring the liquid from B to C or D or E, does not increase or decrease the quantity; the "amount" of liquid remains the same (or is conserved) whether it is in B or in C. Since the quantities A and B were equal, and since pouring the liquid of B into C does not change its quantity, then the quantities in A and C must also be equal. If the child does not consistently assert this equality, then he has failed to conserve.

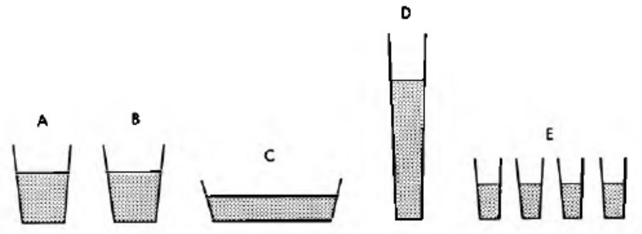

FIGURE 11
Conservation of continuous quantities.

In the case of *conservation of substance,* the child is presented with two identical balls of Plasticine (or clay, etc.). He is first asked whether there is the same amount of Plasticine in both balls. If he does not think so, he is asked to take away or add some clay to make them identical. Then, the experimenter changes one of the balls to a sausage shape, while the child watches. The child must now decide whether or not the ball and the sausage have equal amounts of substance. As in the liquid situation, the ball is changed into a variety of different shapes. If the child consistently asserts that the belli and the new shapes do have equal amounts of substance, then he has conserved substance and has recognized that merely changing the shape does not alter the amount of matter involved.

To test the *conservation of weight,* the experimenter again presents the child with two identical balls of Plasticine and places them on a balance. The child sees that the two balls weigh the same. Then they are removed from the balance and one ball is transformed into the shape of a sausage. The child is asked to anticipate the results of placing the ball and the sausage on the two sides of the balance. Will they still remain balanced or will one side be heavier than the other? The question is whether the child recognizes that weight is conserved despite changes in shape. Here again a series of changes are made to one of the balls and the question as to the identity of weight is repeated.

In the case of *conservation of volume,* two balls of Plasticine are placed in two identical beakers, each filled with equal quantities of liquid. The child sees that the balls displace an equal volume of liquid in both beakers. Or, in the child's terms, the liquid goes up an equal distance in both cases. Then the balls are removed from the beakers, and one ball is changed into the shape of a sausage. The question now is whether the child recognizes that both ball and sausage continue to displace equal volumes, or whether the water goes up an equal amount in both cases.

All these conservations are similar. They involve a first phase in which the child must recognize that two amounts—liquid quantity, substance, weight, or volume—are equal. Most children above the age of 4 years are quite successful in this task. All the conservations also involve a visible transformation which may be done by either the child or the experimenter. While the child watches, or as a result of his own actions, the liquid is poured from one beaker to another, or the ball is changed into a sausage. It is quite apparent that no liquid or Plasticine is added or taken away. It is also apparent that things now *look* different. The column of liquid is shorter and wider, and the ball is now a sausage. And, finally, all the conservations involve a second phase in which the child must once again judge whether the amounts in question are still the same. Of course, they are equivalent, and the issue is whether the child will recognize this or be misled by the observed changes in appearance.

Piaget's general findings are that there is a sequence of development with regard to each of the conservations. Children begin by failing to conserve and require a period of development before they are able to succeed at the task. For example, in the case of continuous quantities, children are not able to conserve until about the age of 6 or 7 years. In the first phase of the problem (two identical beakers, each filled with equal amounts of liquid), the youngest children, around 4 or 5 years of age, correctly conclude that the amounts of liquid are equal. Since the child has either poured out the liquid into the second beaker, or has told the experimenter when to stop pouring, this is not surprising. If asked to justify the identity, the child will say that the water comes up to the same level in each glass so that the amounts are equal. When the liquid in one beaker is poured into a third glass which is different in shape from the first two, the child now maintains that the amounts are no longer equal. One glass has more to drink than the other. Asked to explain his answer, he says that the glass with the taller column of liquid has the greater amount. This judgment of amounts is tied exclusively to the heights of the columns of liquid: when the heights are the same (as in phase 1), the child thinks that the amounts are the same; when they are different (as in phase 2), then the amounts must be different too.

In stage 2, the child of 5 or 6 years vacillates in his responses to the conservation problem. While he usually fails to conserve, his approach to the problem varies from time to time. In the second phase of the experiment (when one beaker is shorter and wider than the other), the child sometimes says that the taller beaker has more to drink, and sometimes maintains that the wider one has the greater amount. Unlike the stage 1 child, he does not concentrate exclusively on the heights of the columns of liquid, but sometimes bases his judgments on the widths as well.

In stage 3, the child is capable of conservation. When asked why the amounts do not change after the pouring, he gives at least one of several reasons. One is that if the liquid in C were returned to its original container, B, then the two initial beakers, A and B, would contain identical columns of liquid. This is the *negation* argument. A second reason is the *identity* argument: it's the same water. You haven't added any or taken any away. A third argument, involving *compensation* or *reciprocity,* is that the third glass, C, is shorter than the original beaker, A, but what C lost in height was compensated by C's gain in width; therefore, the amount in C must be equal to the amount in A.

Toward the end of his life, Piaget returned to the problem of conservation and stressed the role of *commutability.* In one experiment, Piaget (1979) presented children with a conservation of substance problem of the following type. A ball of clay is presented and then a piece is removed. The child is asked if the ball has the same amount, and says no, since something has been taken away. The piece that had been removed from one side of the ball was placed on the other side and the child was again asked if the ball has the same amount now (with the piece added to the other side) as did the original ball. Piaget finds that under these conditions, children assert conservation at a very young age. They say essentially that "It's the same thing, you took it away and then put it back and it's always the same" (p. 21). In other words, the children have understood "that there is displacement, and that when one displaces, what is added at one place has been taken away from another place" (p. 21). This Piaget calls "commutability" and claims that it is one important factor in conservation. Commutability bears a similarity to the notion of compensation.

In the case of conservation of substance, weight, and volume, a similar progression to that of quantity appears. In the first stage, the child fails to conserve apparently because of a concentration on only one of the stimulus dimensions involved. That is, in the case of weight he may say that the sausage is heavier than the ball because the former is longer. In the second stage, he again fails to conserve, although now he vacillates between the two dimensions involved. For instance, he may sometimes believe that the ball is heavier because it is wider and at other times assert that the sausage is heavier because it is longer. In the third stage, the child conserves, for reasons similar to those cited for continuous quantities.

While all the conservations follow a similar course of development, there is a striking irregularity as well—the phenomenon of *horizontal décalage.* This refers to the fact, which has been well substantiated, that the child masters the conservation of discontinuous quantity and substance at about age 6 or 7; does not achieve stage 3 of the conservation of weight until age 9 or 10; does not understand the conservation of volume until approximately 11 or 12. In each case the arguments used are the same, sometimes even involving the same words. But having mastered conservation in one substantive area, like substance, the child is not able to generalize immediately to another area like that of weight. First, he acquires conservation of discontinuous quantity and substance, and then weight, and then volume. The décalage, or lack of immediate transfer, illustrates how concrete is the thought of the child during the ages of about 7 to 11 years. His reasoning is tied to particular situations and objects; his mental operations in one area may not be applied to another, no matter how useful this might be.

GENERAL CHARACTERISTICS OF THOUGHT

We have reviewed the development of various aspects of thought: classes, relations, number, and conservation. It would seem useful at this time to take a broader look at some general characteristics of cognitive development.

Underlying Patterns of Thought

There are striking regularities in the child’s cognitive development. In each of the two major periods of development discussed in this chapter (preoperational and concrete operational), the child uses distinctive patterns of thought to approach different substantive problems. There appear to be some general patterns which characterize the thought of the preoperational child and some other patterns manifested in the concrete operational child’s cognition.

Consider, first, the child from about 4 to 7 years in the preoperational period. (Remember that this age designation is only approximate, since a child as old as 9 or 10 years typically shows a preoperational approach to the conservation of volume.) One general characteristic of cognitive activity during this period is *centration.* The child tends to focus on a limited amount of the information available. In the conservation of number, he judges two sets equal when they are the same length and ignores another relevant variable, the density. In the conservation of continuous quantity, the child judges two amounts equal when the heights of the columns of liquid are the same and ignores the width. In the construction of ordinal relations (the problem of ordering ten sticks in terms of height), he succeeds only by considering the tops of the sticks and ignoring the bottoms, or vice versa. In all these problems, the preoperational child deploys his attention in overly limited ways. He focuses on one dimension of a situation, fails to make use of another, equally relevant dimension, and therefore cannot appreciate the relations between the two. (The notion of centration is somewhat similar to Piaget's earlier concept of juxtaposition which is the tendency to think in terms of the parts of a situation and not integrate them into a whole.)

By contrast, the concrete operational child is characterized by *decentration.* He tends to focus on severed dimensions of a problem simultaneously and to relate these dimensions. In the conservation of number, he coordinates length *and* density: two sets have the same number when the first is longer then the second but the second is denser than the first. In the conservation of continuous quantity, he recognizes that amounts are equal when one column of liquid is at the same time taller but narrower than a second. In the construction of ordinal relations, he determines whether a given object is simultaneously bigger than some objects and smaller than others. In all these problems, the concrete operational child attends to severed aspects of the situation at once. Centration and decentration are general patterns of thought, underlying structures.

The two major periods of development can be characterized in other ways as well. The thought of the preoperational child is *static* in the sense that it centers on states. In the conservation of substance he focuses on the shape of Plasticine (sometimes a ball and sometimes a sausage) and ignores the transformation, that is, the change from one state to the other. In the conservation of continuous quantity he focuses on the heights of the columns of liquid and not on the act of pouring. He lacks adequate representations of an object's shift from one position to another. In general, he concentrates on the static states of a situation and not on its dynamic transformations.

The concrete operational child, on the other hand, is attuned to changes. In the conservations he concentrates on the transformation: the act of pouring the liquid, or spreading apart a set of objects, or deforming a ball into a sausage. He forms more or less accurate images of the changes which have taken place, and, therefore, can reason, for example, that as a set expands in length it simultaneously decreases in density.

The preoperational child's thought lacks *reversibility.* He may be able to predict an empirical reversibility as, for instance, in the case of the liquids where he would agree that if the water were poured back into B, there would be the same quantity as before. But this empirical reversibility does not change the fact that now he believes there is more (or less) water in the new glass C. It is as if pouring from B to C, and from C to B were totally unrelated actions. The older child, on the other hand, realizes that pouring from C to B reverses or negates the action of pouring from B to C and is aware that it is the same action performed in another direction. By carrying out the action mentally, that is, by reversing the pouring in his mind, he is able to ascertain that the quantity of water in C (the lower wider glass) is the same as in B. He can perform a mental operation which leads him to a certain conclusion, and then do the reverse of this operation which enables him to return to his original starting point.

The concrete operational child can also perform another type of reversibility when operating on relations. This is reciprocity. For instance, in the example of liquid quantity, when the child says that one glass is longer and thinner, whereas the other is shorter and wider, he is canceling out the differences between the two glasses by an action of reciprocity. One difference balances out the other, with the result that they have a reciprocal relationship.

To summarize, the preoperational child's thought is irreversible and attentive to limited amounts of information, particularly the static states of reality. The concrete operational child focuses on several aspects of a situation simultaneously, is sensitive to transformations, and can reverse the direction of thought. Piaget conceives of these three aspects of thought— centration-decentration, static-dynamic, irreversibility-reversibility—as interdependent. If the child centers on the static aspects of a situation, he is unlikely to appreciate transformations. If he does not represent transformations, the child is unlikely to reverse his thought. By decentering, he comes to be aware of the transformations, which thus lead to reversibility in his thought. In conclusion, we can see that one aspect of thought is not isolated from the rest. Even though the nature of the system may vary with the development of the child, thought processes form an integrated system.

Invariant Sequence

Another striking regularity in cognitive development involves *invariant order:* the sequence of activities (for example in classification, partial alignments, collections, class inclusion) assumes an invariant order despite wide variations in culture. Cross-cultural research provides relevant evidence on this issue. Within Western cultures children progress through the various stages in the order described by Piaget. In the case of conservation of continuous quantities, for example, research shows that Swiss, British, American, and Canadian children first fail to conserve, then vacillate in their response, and later conserve with stability. While children in these cultures do not necessarily achieve the various stages at the same average ages, the sequence of development—the order of the stages— seems identical in all cases. Even in other and very different cultures, like the Thai or Malaysian, the same sequence of stages and type of responses appear. Children in Thailand, for example, exhibit classification activities which are virtually identical to those used by Western children, and proceed through the sequence of stages in the order described by Piaget (Opper, in Dasen, 1977). There is great cross-cultural generality in Piaget's findings. At the same time, we must make one qualification: apparently, members of some cultures do not advance as far in the sequence of stages as do Westerners. Thus, for whatever reasons, in some cultures, individuals may not complete the stage of formal operations. Not everyone achieves the highest level possible in terms of Piaget's stages. Yet, until their progress terminates, these individuals proceed through the sequence of stages in the standard order. While the ultimate level of development may differ among cultures, the sequence seems to be invariant, as Piaget proposes. The phenomena described by Piaget are thus nearly universal, occurring across extreme variations in culture and environment. Piaget has surely captured something very basic in human cognition.[6]

Irregularities

Piaget has gone to great lengths to dispel some misinterpretations concerning his theory. In particular, he shows that there are certain *irregularities* in development. He points out, first, that the ages at which the stages occur vary considerably both within and among cultures. Not all Genevan children attain stage 3 of number development at 6 or 7 years, and children in Martinique lag behind Genevans by approximately four years. In Thailand, urban children attain stage 3 at the same time as children in Geneva, but rural Thai children lag behind by approximately three years. In Malaysia, rural children attain the number concept one year ahead of urban children, who in turn lag behind Swiss children by two years. Thus the rate of development seems to vary from group to group. Second, the course of an individual's development is continuous.

The child is not characterized by stage 1 one day and by stage 2 the next day. Rather, the transition is gradual, occurring over a long period of time, and the child exhibits many forms of behavior intermediary between the two stages. Indeed, an individual child's behavior takes many forms in addition to those Piaget describes as being typical of the various stages. Piaget's stages are idealized abstractions; they describe selected and salient points on an irregular continuum of development. Third, the child is not always in the same stage of development with regard to different areas of thought. The child may be characterized by stage 2 in the case of classes, and stage 1 in the case of relations. It is unlikely, however, that he will be in stage 1 for classes and stage 3 for relations. Only infrequently does one find extreme discrepancies between stage levels in different areas. Fourth, as we have already seen, there exists the phenomenon of *horizontal décalage,* in which the child displays different levels of achievement in regard to problems involving similar mental operations; for example, he may be able to conserve substance but not number.

Preoperational Strengths

Piaget *(On the Development of Memory and Identity,* 1968) tries to correct a widespread misconception concerning preoperational thought. Typically, we characterize the young child as intellectually *incompetent* since he cannot conserve, cannot use reversibility, and cannot decenter. Piaget feels that this view is exaggerated; as a result of recent research, Piaget proposes that the preoperational child possesses a number of important intellectual strengths which must not be overlooked. In particular, the young child is capable of *identity, functions,* and *correspondences.*

While unable to conserve, the young child nevertheless appreciates certain basic identities. For example, in the standard conservation problem, the young child recognizes that the *same liquid* is transferred from one beaker to another even though one looks quite different from the other. He sees that the basic substance does not change, even though its appearance is altered and even though he falsely believes that the *amount* of liquid has changed. He appreciates identity but fails to conserve quantity.

Piaget proposes that the notion of identity may derive from the child's perception of his own body's growth. With Gilbert Voyat, Piaget asked children to draw themselves when they were babies, when they were a little bigger, and so on; then the experimenters questioned the children concerning the maintenance of their identity despite obvious physical changes. The experimenters also posed similar questions concerning the identity of other objects, including plants. The results showed that children easily appreciated their own identity despite changes in size, and were less likely to accept the continuing identity of a plant over its various changes in appearance. Perhaps, then, the notion of identity derives from the child's perception of his own body's growth and later is generalized to the world of objects.

The preoperational child can also perceive functional relations in the environment. One example of such *functions* (given by Sinclair, 1971) involves the opening of a curtain: "the child understands that when one pulls the cord of a curtain, the curtain opens; the farther one pulls, the farther the curtain opens" (p. 4). In other words, there is a functional relation, a co-variation between pulling and opening, and the child perceives that the two factors are positively related. (There may even be precursors of functions in infancy: this example is reminiscent of the infant Laurent who seemed to realize that the more vigorously he shook a chain, the louder would be the sound produced by the attached rattles.) It is very important, of course, for the child to recognize such functional relations in the environment: they pervade it. The taller the person, the stronger he is likely to be; the harder one hits another child, the more likely is the child to protest and even cry; the bigger the glass, the more milk it holds. Despite limitations in other areas of thought (for example, centration), the preoperational child has some appreciation for basic functional relations, and this is of great value to him in coping with the environment.

At the same time, Piaget points out that these functions are incomplete: they constitute only a *semilogic.* For one thing, the child's appreciation of functions is imprecise. To return to the example of the curtain, the child does not realize exactly how the pulling of the cord is related to the opening of the curtain and cannot quantify the results with any degree of precision. Another Piagetian experiment makes this clear. Children were presented with three toy fish, 5, 10, and 15 centimeters long, respectively, and were told to feed each fish its proper diet of meatballs. The middle-sized fish should get twice as many meatballs as the smallest, and the largest fish three times as many. Preoperational children understood the functional relation between size of fish and number of meatballs only in an imprecise way. They realized that the larger the fish, the more it needs to eat. But they were not able to work out the function in a precise manner (for example, by giving 2, 4, and 6 or 3, 6, and 9 meatballs to the respective fish).

Toward the end of his life, Piaget (1979) stressed the role of "correspondences." He used this notion to refer to the child's tendency to compare objects or events, to determine the ways in which they "correspond," or are similar and different. This tendency appears at all levels of development, from infancy onward, although it takes different forms at different levels.

For example, an infant first hits a toy parrot to make it swing and then applies the hitting scheme to other hanging objects as well. In a sense he has compared the new object with the familiar parrot and noted the similarity between them (the correspondence of one object to another).

Note: A black marble and a white marble are glued to a plate, with the white one above the black one (as in Figure 12A). Then the plate is rotated so that black one is above the white one (as in Figure 12B).

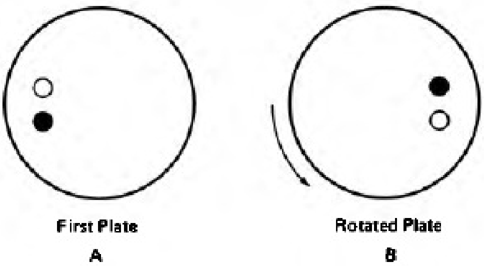

FIGURE 12
Correspondence of Marbles.

The preoperational child displays different forms of correspondence. For example, in one experiment, Piaget (1979) showed children two objects on a rotating disk. Imagine that the objects are a white marble and a black marble, glued to a dish, as in Figure 12A. When the marbles are on the left (Figure 12A), the white is above the black. When the dish is rotated so that the marbles are on the right (Figure 12B), then the black is above the white. The preoperational child observes the situations—the marbles on the left and on the right—and gradually notes the correspondences between them. The child sees that when the marbles are on the left side, the white is higher, but when they are on the right, the white becomes the lower. At first, the child's approach is simply "empirical": to record the facts without interpreting them. But "the child discovers suddenly that there is a general order" (p. 24). He determines, in other words, that there is a reversal of position. It's not just that the white is higher in one situation and lower in the other, but that the white has *switched positions.* This insight then gradually leads the child to another: the positions were switched because a transformation took place. The rotation of the dish caused the switch in position.

We see then that the child begins by comparing two states, noting some basic similarities and differences (the switch in position). These correspondences are important because they pave the way for the child's appreciation of transformations. And as we have seen, an appreciation of transformations is at the heart of concrete operational thinking.

In brief, preoperational thought is not characterized solely by incompetence. Young children appreciate certain basic aspects of identity, perhaps as a result of experience with their own bodies. They also understand, albeit in an imprecise manner, various simple functional relations in the environment. They detect correspondences, and this leads them to an appreciation of transformations. In dealing with young children one must be aware of these strengths as well as of the commonly cited limitations, as Gelman and Gallistel (1978) and other contemporary writers concur in maintaining.

The Concept of Stage

Piaget's theory describes a sequence of *stages.* For example, in the case of the conservation of number we have reviewed the transition from centration to decentration. Now it is important to consider the nature of such stages. What does Piaget mean by *stage* and how useful a concept is it?

According to Piaget (*Biology and Knowledge,* 1971a, p. 17) the notion of stage is used when the following three conditions are fulfilled. First, there must be an *invariant sequence* of activities. Thus, in the case of conservation, there is, first, a failure to recognize equivalence; then there is vacillation; and, finally, there is success. The order of appearance of the activities is the same for all children. Second, each stage in the sequence is characterized by an *underlying structure,* a core system determining the child's overt behavior. Thus, underlying the child's failure to conserve is the strategy of centration—the tendency to focus on limited amounts of information. Third, each of the structures *prepares the way* for a succeeding one. Thus, in the case of conservation, the initial centration prepares the way for a vacillation among the available dimensions, and this in turn leads to the subsequent decentration. In brief, Piaget proposes that stages are characterized by invariant sequence, underlying structures, and successive integrations.

Piaget also emphasized that despite the existence of stages, development is *continuous.* The child does not enter a new stage overnight; instead, the changes are gradual, and indeed barely perceptible from close-up. Piaget explained this in terms of the *scale of measurement.* If we look closely at a child's development, observing every day and thus using a fine scale of measurement, it is hard for us to see dramatic changes; from one day to the next we will not notice differences in stages. But if we stand back, observing the child infrequently and thus using a crude scale of measurement, we will be impressed with changes; from one year to the next we will see progress from one stage to the next.

We have already reviewed research concerning the notions of invariant sequence and underlying structure. Cross-cultural study demonstrates that the sequence described by Piaget is extremely widespread, if not universal. Also, there seem to be distinct underlying patterns or structures in each of the major periods under consideration—preoperational and concrete operational. Consider next Piaget's third condition for the existence of a stage—the requirement that each stage prepare the way for the next. While it is hard to adduce evidence supporting this notion, it seems to have a certain amount of face validity; for example, a focus on two dimensions seems naturally to follow from a focus on one. In brief, the evidence concerning invariant sequence, underlying structures, and successive integrations seems to support Piaget's proposition concerning the existence of major stages of development.

At the same time, the stage notion suffers from a number of difficulties. One, already alluded to, is the existence of irregularities in development. We have seen that the child is not always in the same stage with regard to different areas of thought. Thus, he may be in stage 1 with respect to classes and stage 2 in the case of relations. Also, the phenomenon of *horizontal décalage* is very striking: the child may display different levels of achievement in regard to very similar areas of thought. Thus, he may conserve substance but not number. The existence of these irregularities seems dissonant with the notion of distinct underlying patterns or structures of thought characterizing the major stages of development. If the patterns are so strong and pervasive, why are the *décalage s* so striking?

Another difficulty with the stage notion is that the structures presumably underlying a stage may also be implicated in stages occurring earlier in the sequence. Thus we have recent evidence by Trabasso (1975), for example, to the effect that under certain conditions, preoperational children can perform concrete operational tasks. If the same structures underlie behavior at different stages, do we not then have to alter our notion of stages? The issue of stages is extremely complex and is now the subject of considerable rethinking (for an excellent discussion see Flavell, 1985).

Indeed, toward the end of his life, Piaget seems to have rethought the stage notion himself. The last ten years of Piaget's research revolved largely around issues of cognitive change and development and did not employ stage notions to any significant degree. In this sense, Piaget became less of a "structuralist" (one who deals with the analysis of mental structures underlying the stages) and more of a "functionalist" (one who deals with the factors determining development). As we shall see in Chapter

6, Piaget's theory of equilibration placed the emphasis on gradual changes or in effect on many fleeting substages. What was important for the later Piaget was not a concept of broad, stable stages, but a theory of the continuous change and development of the child's intellectual structures.

MENTAL IMAGERY

After his brief examination during the 1920s of the *content* of thought, Piaget's main concern has been with the *operative* aspect of cognition. This refers to *actions* used to deal with or even change the world. These actions may be either overt or internal. Examples of overt actions abound in the sensorimotor period. The infant kicks to shake a rattle, or uses a stick to draw an object close. The present chapter has covered two major subdivisions of internalized actions: the isolated and unrelated actions of preoperational thought and the structured and coordinated ones of concrete operational thought.

Piaget has also shown an interest, albeit a lesser one, in the figurative aspect of cognition. This refers to three ways in which the child produces an account of reality. One is perception, a system which functions by means of the senses and operates on an immediately present object or event. It is through perception that the child achieves a record of the things in the surrounding world. This record is often inexact, as in the case of the visual illusions. A second subdivision is imitation, by which the child reproduces the actions of persons or things. It is true that imitation involves actions on the part of the child, but these actions nevertheless fall under the figurative aspect since they produce a copy of reality but do not modify it. A third portion of the figurative aspect is mental imagery. As we saw in Chapter 3, mental imagery refers to personal and idiosyncratic internal events which stand for or represent absent objects or events. When we "picture" to ourselves our first bicycle, or the stroll we took last week, then we are using mental imagery. As we see from this last example, the topic of memory is closely bound up with the figurative aspect of thought. Memory (recall) typically involves retaining knowledge gained through the figurative mode.

In recent years, Piaget has conducted important investigations into two important aspects of figurative cognition, specifically imagery and memory. His theory stands in stark contrast to the traditional *empiricist* view of these matters. The latter assumes that perception stamps on the individual a literal copy of reality. Given sufficiently frequent repetition of the initial event, a mental image mirroring the reality is formed and is stored in memory. If there is no further experience with the original event, the memory image gradually fades, losing its fidelity to the reality; it is forgotten. Piaget criticizes this traditional view on several grounds. Most important, he believe that reality does not simply impose itself on a passive organism. Rather the individual assists in the construction of his own reality. His intellectual activities—the operative mode of thought—serve to shape the results of encounters with the environment. The resulting figurative knowledge is not simply a copy of reality. This theme—the influence of operative structures on figurative knowledge—dominates Piaget's discussion of mental imagery and memory. We will now consider these two topics in succession.

History

Mental imagery was one of the first topics studied by experimental psychologists. At the end of the nineteenth century, the school of Wundt used the introspective method to analyze the nature of mental imagery. The Wundtians believed that images were composed of a bundle of sensations tied together by means of association. At the beginning of the twentieth century, the study of imagery fell into disrepute for two reasons. First, the Wurzburg psychologists found that much of thought did not seem to involve imagery at all, and second, the behaviorist revolution which occurred in the United States maintained that the introspective method was a poor one. The behaviorists felt that the data of introspection—one's impressions of one's own consciousness—were not public enough. How could another psychologist determine if an introspection were reliable and accurate? As a result of the behaviorist attack on the method of introspection, the study of imagery was considered "unscientific" and was largely abandoned. Recently, however, psychologists have shown a renewed interest in the ancient problem of imagery, and the topic is once again becoming central to experimental psychology (Neisser, 1976).

In contrast to modern investigators, Piaget has been studying imagery since at least the 1930s. In Chapter 3 we discussed Piaget's work on imagery in the young child up to the age of 4 years. If you will recall, this theory proposed that mental images do not occur until about the middle of the second year. Before this time the child did not possess mental representations of the environment and, as a result, reacted mainly to events occurring in the present. After imagery makes its appearance the child can represent to himself both events that occurred in the past and objects that are no longer perceptually present. Also, according to Piaget's theory, imagery results from imitation. At first, the child overtly imitates the actions of things or people; later, his imitation becomes internalized and abbreviated. It is through this internal activity that images arise. Clearly, Piaget's views contrast strongly with Wundt's. Images are not merely bundles of sensations, imposed by the environment and connected by association; rather, the construction of images involves the activity of internalized imitation.

Later, with Inhelder, Piaget returned to the study of imagery (1971). His later work deals with children above the age of 4, and poses a number of interesting questions. For example, are there different types of images at different stages of intellectual development? If so, what is the relation between the images and the mental operations of a given stage?

Method

While these questions are interesting, the study of mental images is very difficult, especially in the case of children. Images are personal, idiosyncratic events which cannot be viewed directly. One cannot "see" another person's imagery; the investigator must, therefore, infer their existence and nature from other phenomena, such as a verbal report. Piaget has used a variety of methods to study imagery. One of these methods is to ask a person to describe his own images. But language is not fully adequate for this task, or even for describing something as concrete as the immediate perception of an object. We are never able to convey by words the precise nature of what we see. In our attempt to describe percepts, we inevitably emphasize certain features and neglect others. We have difficulty in describing shades of colors, or gradations of textures. We cannot give an impression of the entire percept at once, but must describe its details in sequence, and thereby often lose the essence of the whole. If language so poorly conveys perceptual events which continue to remain before our eyes for further inspection, how much more difficult is it to describe mental images which often are fleeting and unstable?

Another method of studying mental images is by drawing. Here the person is asked to draw an object previously presented. Since the object is no longer present, he must produce an image of it to yield the drawing. The drawing, therefore, gives some insight into the nature of the image, which is the internal "picture" of the object. The method of drawing, however, presents several shortcomings. Drawing is not a simple and direct reflection of images; it also involves other processes. Some persons have poor memory. If they have forgotten their image of an object, they cannot very well draw it. Other persons simply cannot draw well. It is not their image that is at fault, but their artistic skill.

A third method attempts to bypass the shortcomings of original drawings. The subject is given a collection of drawings made by the experimenter, and must select from them the one most closely corresponding to his image of what he had previously observed. This method, of course, is not affected by variations in subjects' artistic abilities and reduces the difficulties created by a poor evocative memory. But even the method of selection from a collection of drawings is not altogether satisfactory. One problem is that the drawings presented are not likely to be exact copies of the person's mental image. The drawings may omit details of the original image or add new features. In either event, the subject's choice does not give a fully accurate indication of his image.

To study imagery, Piaget has used all these methods—verbal report, drawing, and selection of drawings—either alone or in combination. As is customary with the explorations carried out by the Geneva school, the methods were supplemented by verbal questioning carried out in the clinical manner.

Major Findings

One experiment was concerned with *kinetic images,* or the imagery of an object's movement. Children from about 4 to 8 years of age were presented with two identical blocks, one on top of the other (see Figure 13A). Each subject was asked to draw the situation, and generally did this quite well. Then the top block was moved so that it slightly overlapped the bottom one, as in Figure 13B. After the child had had a chance to look at this for a while, the top block was returned to its original position (Figure 13A). The child was then asked to draw the block in its displaced position (Figure 13B), which was, of course, no longer visible. After this, a collection of drawings was presented. This contained a correct rendering of Figure 13B as well as an assortment of incorrect drawings which represented errors typically made by children of this age. (This technique is similar to the use of countersuggestions in the interview.) The child was asked to select the drawing which he felt corresponded most closely to what he had seen. In the final step another control was added. The top block was once again displaced, and the child was asked to draw the situation while it was present. If the child could accurately draw the blocks when present, then any of his previous errors of drawing (when the blocks were absent) must be due to faulty imagery or memory and not to faulty drawing ability.

FIGURE 13
Movement of blocks.

To summarize, the child First drew the displaced blocks after they were no longer visible; then he selected from a group of drawings one resembling the displaced blocks; and finally, when the displaced blocks were once again before him, he drew them.

The findings show that before the age of 7 years, children can draw the displaced blocks quite correctly when they are present, but not when they are absent; nor can the children choose a drawing which corresponds to the situation. In general, children of about 4 and 5 years produced and selected drawings of the types A through E (see Figure 14), whereas children of 6 years made errors like those of types F and G. It was only at 7 years that over 75 percent of the subjects both drew and chose the correct drawings.

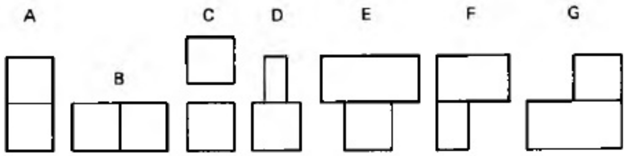

FIGURE 14
Drawing of blocks.

A cross-cultural study of this problem in Thailand (Opper, 1971) shows that Thai children make the same types of errors as do Swiss children, although it is not until 10 years of age that 75 percent of the Thai subjects make correct drawings of the two blocks.

The responses of the younger child would seem to indicate that he forms only a very general picture of the situation, that is, that one block has been moved. When asked to draw the exact details, he is unable to do so. The child therefore reproduces this general impression of movement by detaching the top block from the bottom (cf. C), by a symmetrical movement of shrinking or enlargement of one of the two blocks (cf. D and E), or, finally, by the retention of one common boundary or identical line for the two blocks, in addition to making changes on the other side of the blocks (cf. F and G). His image does not appear to correspond to the actual situation. The child seems to center on one dimension, that is, on one particular aspect of the situation—for example, the overlapping of the top block in drawings E and F, or the overlapping of the bottom block in drawings D and G. However, the child does not coordinate the movement of one block with the final state of the two blocks. Apparently the child does not analyze the situation in sufficient detail but merely forms a global impression of what has happened. He is aware that the block has moved, but the intimate details of the movement and the ensuing displacement seem to have escaped his attention. As a result, his mental image is inadequate.

A second type of imagery is *static* imagery. In this instance the image reproduces a collection of objects, a scene, or a picture—in brief, any situation in which the elements remain unchanged in either shape or position. Piaget finds that the child is able to produce adequate static imagery earlier than kinetic.

We have reviewed only a small sampling of Piaget's experiments on imagery. Their results, together with those of a great many more studies, have led Piaget to draw the following general conclusions concerning imagery and its relation to intelligence as a whole. First, imagery develops in a gradual manner. The evolution of imagery is not as dramatic as that of the cognitive operations which display a clear-cut sequence of stages. There appears to be only one major turning point in the development of images. This seems to occur at around the age of 7 or 8 years and corresponds to the onset of the period of concrete operations. Before the break, that is, from the age of 1 1/2 to about 7 years, the child seems capable of producing with any degree of accuracy only static images, and even these are far from perfect. The child cannot represent correctly the movements of an object or even simple physical transformations; the images produced for such situations are grossly deformed.

Piaget believes that the reason for this deficiency is one aspect of operative cognition, namely, a tendency to concentrate on the initial and final states of a given situation and to neglect the intervening events which are responsible for the changes. We have already seen this tendency, which is called *centration,* operating in the case of conservation. If you will, recall the situation where the child was presented with a line of vases, each of which contained a flower. The flowers were removed from the vases and spread apart. When this occurs, the young child usually believes that there are more flowers than vases, since the line of flowers is now longer than the line of vases. He has *centered* on the lengths and ignored a number of other factors. He has failed to decenter and to consider the density of the lines, as well as their length, and he has ignored the intermediary transformation (the removal and spacing of the flowers). Thus, the child focuses mainly on the initial and final states (the flowers in the vases and the flowers spaced out) and fails to integrate these impressions with all else that has occurred. Thus, before the age of 7 or 8 imagery is extremely static. As a result, the child produces a distorted picture of reality characterized by an emphasis on superficial features which are each isolated from others and not coordinated into a coherent whole.

From about the age of 7 years onward, however, the child becomes capable of producing images which can reproduce kinetic situations. This improvement is due to the fact that he can now imagine not only the initial and final states, but also the intermediary transformations. His imagery has become less static. Of course, it is never possible to reproduce *all* the intervening events, since in some cases (like the pouring of liquid), they occur rapidly. But the child recognizes that a sequence is involved and that there has been a series of intervening steps between the initial and final states.

A final question concerns the relation between dynamic images and the concrete operations. Kinetic images occur at approximately the same time that the child becomes capable of the concrete operations; what then is the relation between the operative and figurative aspects of thought at this stage? On the one hand, we have already seen that operative cognition influences the nature of the child's imagery. Thus, the concrete operational child's decentration contributes to the dynamic nature of his imagery. In Piaget's theory, figurative cognition (here, imagery) is dominated by operative cognition (here, the concrete operations). On the other hand, images can play an auxiliary role in thinking. For example, consider the number conservation task involving flowers and vases. The concrete operational child can form accurate transformational images of the displacement of the flowers. After the transformation has been done, he correctly pictures the way in which the flowers have been removed from the vases. The ability to form images of this sort does not *guarantee* that the child can conserve number; as we have already seen, the processes underlying conservation are not solely perceptual or imaginal. Nevertheless, the child who has a correct image of the transformation is certainly ahead of the child who does not. In other words, images are a useful and necessary auxiliary to thought during the concrete operational stage. By providing relatively accurate representations of the world, images assist the process of reasoning although they do not cause it.

Summary and Conclusions

Images represent absent objects or events. They are “symbols,” in the sense of bearing some resemblance to the object represented, and are personal and idiosyncratic. Images do not give as complete and detailed a reproduction of the object as is provided by direct perception. Images first make their appearance around the middle of the second year of life, and they arise from a process of imitation which gradually becomes internalized. Until the age of approximately 7 years, the child is only able to produce approximately correct mental images of static situations. He concentrates on states rather than on transformations. The limited imagery of the child is partly the result of immature operative structures. As these structures develop, so does his imagery. After the age of about 7 years, the child becomes capable of correct kinetic imagery. This new ability permits a further understanding of reality: the child now has available a more accurate and detailed rendering of the events on which to focus his reasoning.

MEMORY

Memory, too, is influenced by operative cognition. Before exploring this, it is necessary to begin by clarifying some terminology.

Definitions

In ordinary language, we use the words “memory” or “remember” in several different senses. Here is an anecdote to illustrate the point. An adult has not ridden a bicycle since childhood, some years ago. Now his own child gets a bicycle and asks whether the adult “remembers” how to ride. “Of course, I remember how to ride a bicycle,” says the adult. Asked (skeptically) to prove it, the adult get on, and pedals around a bit. Despite the lack of practice over a long period of time, he is able to ride very smoothly, much to the surprise of the child who owns the bicycle and who now wonders whether he will get to ride it. As the adult is pedaling down the street, he “remembers” riding the bicycle which he owned as a child. He has a fairly clear mental picture of its overall shape and form, as well as the places in which he rode.

This example illustrates two very different kinds of "memory." In the first kind, the adult *remembers how to do something.* Although there has been no practice for many years, he has not lost general bicycle-riding skills. He "remembers" how to ride not just a specific bicycle, but any bicycle. Through experience, he has acquired a physical skill of a general nature, and remembers it. In this instance, we use the term *memory* to indicate that the past still exerts an influence on the present. The adult's ability to ride a bicycle, acquired through a set of earlier learning experiences, was somehow preserved within him. Note that after childhood this ability existed as a potential, since until this incident he did not actually engage in the behavior. Note, too, that the element of earlier learning is crucial to the definition. It would not make sense to say, "I remember how to sneeze," since sneezing was never learned. Yet it *would* make sense to say, "I remember how to keep from sneezing" since that *was* learned. In brief, this is one valid use of *memory:* a person can retain, over a period of time, a behavioral potential which is the result of previous learning.

The other sense of *memory* is quite different. When the adult "remembers" riding his childhood bicycle, he is referring to a specific event and thing in the past. He has a hold on a particular slice of his own history. He "remembers" a bicycle with wide tires, and a heavy frame—a Schwinn, in fact. He remembers riding it up Commonwealth Avenue to a park with a certain kind of path. This kind of *memory* is more specific and concrete than the first. In this kind of remembering, the adult retains specific events or things from the past; in the other kind of remembering, he preserves the general skills acquired in the past. Often the two types of memory occur together. A person remembers how to type (thus preserving the general ability) and also remembers the specific typewriter used in his early lessons (thus retaining information concerning a specific thing from the past). But the two types of memory do not have to coexist. A person may remember how to type and yet may have totally forgotten the specific typewriter or his early lessons. Similarly, a person may remember the typewriter and lessons, but not remember how to type. Thus, we have used some examples of physical skills to illustrate a distinction between two types of memory.

In the intellectual domain, Piaget's theory (Piaget and Inhelder, *Memory and Intelligence, MEM,* 1973) proposes a similar distinction between "memory in the wider sense" and "memory in the specific sense." The former refers to "the conservation of the entire past, or at least of everything in the subject's past that serves to inform his present action or understanding" (*MEM,* p. 1). More precisely, memory in the wider sense refers to the "conservation of schemes," to the retention of acquired patterns of behavior or thought, like the concrete operations. By contrast, memory in the specific sense "refers explicitly to the past," to specific events or things or persons in an individual's history. Another way of looking at the distinction is to say that memory in the wider sense involves the *operative* aspect of thought: it is the way in which general operations or ways of doing things are preserved over time. Memory in the specific sense is generally *figurative:* it preserves information concerning specific things—a face, an object, an activity. (These "things" include actions, but only specific actions that are thought to have actually occurred, not the potential for actions of a general type.)

Piaget goes on to propose some further distinctions concerning memory in the specific sense. This type of memory—and we shall now simply use the word *memory* to refer to it—may take one of several forms. Perhaps the most primitive is *recognition.* This occurs when a person encounters things (an event, person, thing, etc.) previously experienced and "has the impression of having perceived them before (rightly or wrongly, for there are false recognitions)" *{MEM,* p. 5). Thus, we see someone known before, and "say to ourselves" that the person is familiar, even though his name may elude us and we cannot recall where we knew him. Similarly, the baby in the sensorimotor period recognizes faces and places when they are encountered. Or the baby shows through his abbreviated schemes that he recognizes a toy he has played with. *Recognition,* then, is one form of (specific) memory, involving an impression of familiarity upon an encounter with a previously experienced object.

Recall is a much more sophisticated and difficult form of memory. It involves producing a mental account of a previously experienced thing in the total absence of that thing. One example would be remembering your childhood bicycle or your first grade teacher. *Recall* sometimes involves a mental picture, sometimes words, sometimes an odor. The crucial aspect of *recall* is that the individual produces some kind of mental representation of the previously experienced event.[7] It is evident that recall is closely linked with the semiotic function, already discussed, since the latter involves the formation of mental representations for absent things or events.

The General Hypothesis

Piaget's main interest is in the functioning of memory in the specific sense—recognition and recall. How does specific memory operate?

According to some empiricist views, memory works in the following manner. An individual perceives an object and stores within him its replica or trace. The more frequently or recently the object is perceived, the stronger the trace, and hence the stronger and more accurate the memory. In this classic view, memory is simply a copy of something real, and the accuracy of the copy depends on such factors as frequency, recency, and the like. Note that in the classic view, the individual is mainly passive: things impose themselves on him; they make an impression on him; they form a trace in him as a piece of chalk leaves a record on a slate (hence the expression *tabula rasa,* or blank slate).[8]

Piaget's view is different. He proposes that the child does not simply record reality in a passive manner, storing a copy in the warehouse of memory. Instead, as Piaget sees it, the child *assimilates* and *interprets* reality, so that memory is in part a function of the child's intellectual operations. Memory stems not only from experience but from intelligence. This, then, is the general hypothesis with which Piaget begins his empirical investigations. Given this theoretical framework, Piaget goes on to investigate the specific ways in which mental operations affect memory, especially recall.

Experiments on Memory of a Series

To study the influence of knowing on remembering, Piaget conducted several experiments, one of which involved memory for a series, a topic already reviewed in this chapter. Children of various ages were shown ten wooden sticks, already arranged in a complete series, from smallest to largest. Each child was "told to take a good look at it and remember what he has seen. ' ' Then about a week later, each child was asked to recall the series by drawing it or by tracing it out with his fingers on the table. After this, the experimenter determined the child's stage of development with respect to seriation by giving him the usual tests. The experimenter also obtained a check on the child's drawing ability by having him copy a series of sticks available to direct perception. This copy could then be compared with the child's drawing from memory to determine if distortions in the latter stem from mere drawing deficiencies. In brief, the experiment involved (1) determining children's intellectual level with respect to seriation, (2) presenting them with a completed series to remember, and (3) measuring recall by finger tracing and drawing. Furthermore, (4) a measure of drawing ability was taken so that this factor could be controlled.

What should happen in such an experiment? According to the classic view, the series impresses itself on the passive subject, and the accuracy of recall depends on the extent of the subject's experience with it and on similar factors. The child's drawings should to some degree mirror the reality which impinges on him. Piaget's view is much different: the child actively assimilates the reality into his intellectual system and this process of interpretation determines the nature and quality of recall. In the present instance, a stage 1 child may distort his memory of the series in accordance with his immature intellectual operations, and this will be reflected in his drawing and tracing. Note that the result of this is not a drawing which is simply a pale copy of the reality. Rather, it is a drawing which is systematically distorted in line with the child's intellectual operations.

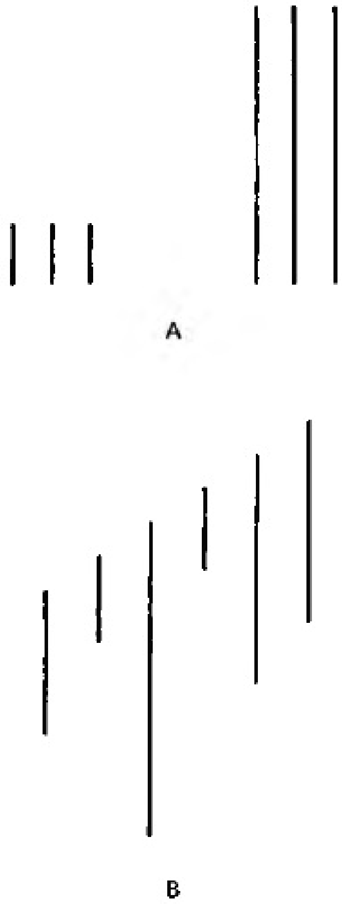

FIGURE 15
Drawings of completed series.

Consider a few examples of this. One child made a drawing like that in Figure 15A, involving several identical long lines and several identical short ones. This drawing was similar to the child's actual arrangement of the sticks during the test of seriation: he made one bunch of large sticks and another bunch of small sticks, but did not accurately seriate within each bunch. Another child produced a drawing like that in Figure 15B. This, too, was similar to his actual arrangement of the sticks. He made the tops of the sticks increase in order of size, but totally ignored the bottoms. (When asked to copy a well-formed series immediately in front of them, these same children produced far more accurate drawings. This allows us to conclude that drawing skill in itself is not at issue.) By contrast, children in stage 3 who could accurately seriate were accurate in recall, as indicated by veridical drawings and tracings.

These findings can be taken to support Piaget's theory. The individual's memory is influenced and organized to some degree by his intellectual operations. *The child recalls not what he has seen but what he knows.* In the present instance, stage 1 children's recall is distorted by their immature seriation schemes. (We shall see cases later where the effect is of a different sort.) At the same time, Piaget points out that the results are not entirely clear-cut. Some stage 1 children make perfectly accurate drawings. Their mental operations do not seem to intervene so forcefully in the act of recall. Instead they seem to focus on the appearance of the series—on its "figurative aspects"—and manage to recall it very well, much as they would recall (and draw) a circle or a tree or a staircase. It is hard to explain why some stage 1 children show the distorting effects of intellectual operations while others do not.

In brief, while there is some variability, the results show that intelligence—the intellectual operations—structures the child's recall. Knowledge interacts with perception to produce what is remembered.

The Development of Memory

According to Piaget, there is a general developmental progression from the early appearance of accurate recognition to the later use of accurate recall. Memory begins in a crude fashion during the sensorimotor period. At this time, the infant shows evidence of recognition. Through overt or abbreviated behavior, he demonstrates that a toy or a person is familiar. The infant does not seem capable of more demanding forms of memory, especially recall (this of course involves evoking a mental representation of absent objects or events). It is only with the onset of the semiotic function, at about 18 months, that the child becomes capable of mental representation and hence recall. Earlier, in another context, we cited the example of Jacqueline, at 1; 11(11), who upon returning from a trip, was able to report on events which had occurred earlier: "Robert cry, duck swim in lake, gone away" *(Play, Dreams, and Imitation,* p. 222). This is an example of recall in a child who is just beginning to give evidence of the use of the semiotic function. In brief, infants show signs of recognition memory, whereas recall, as one aspect of the semiotic function, begins to appear only at about 18 months.

As we have seen, once recall appears, its functioning is influenced by the intellectual operations. Now we shall see that this influence can have developmental aspects. Piaget's experiments on memory for a series shed light on this issue. We already know that the child's recall after one week is distorted in line with his current stage of seriation. But what happens to recall over a longer period of time, say, six to eight months? According to the classic view, the memory trace simply fades, and this fading becomes more complete as time goes on. In Piaget's view, matters are more complex than that. In many cases, there may well be some deterioration of memory over a long period of time. And yet there are other possibilities as well. Memory, which depends on intelligence, therefore exhibits developmental changes which correspond to the development of intelligence. Indeed, Piaget's theory leads to the prediction that under certain circumstances, recall may actually *improve* over time.

In the case of seriation, the matter works as follows: the stage 1 child sees a well-ordered series and assimilates it into his intellectual operations. Since these are immature, one week later the child inaccurately recalls the sticks as a collection of small ones and a collection of large ones. His intelligence has organized recall poorly. Then over a period of time, the child's mental operations develop and he enters stage 3. Now, asked to recall the sticks, he remembers a well-formed series. His memory has improved over time because his intellectual structures have developed more fully.

This is indeed precisely the result which Piaget discovered. Of twenty-four stage 1 children, twenty-two showed improved recall (as measured by drawings) when they advanced to a later stage seven or eight months after the initial testing.

Several comments should be made at this point. First, independent investigators have had a hard time replicating this result (for example, Samuels, 1976). A good deal of careful research, with adequate controls, needs to be done to pin down the effect. It is particularly important to obtain direct measures of the child's assumed intellectual development. Second, it is important to recognize that Piaget's theory does not always predict improvement in long-term recall. Improvement can be expected to occur only if the initial recall was distorted by immature intellectual operations and if these operations subsequently improve. This is a very special case, however, and often does not occur. For example, suppose a child learns someone’s name and tries to recall it a year later. Memory for the name is likely to deteriorate regardless of the child’s stage of development. The child’s advancement from stage 1 to 3 of concrete operations will have no particular bearing on the recall of names, since the recall is merely figurative with no logical operations involved. Here is another example, which may seem paradoxical. Suppose a stage 1 child is shown a badly formed series. After one week he accurately remembers the badly formed series because he has assimilated it into his immature mental operations. Then, over the next year, the child’s mental operations advance and he has reached stage 3. Now when asked to recall the sticks, he produces a well-formed series which is the product of his current intellectual structure. Unfortunately, this is inaccurate recall, since the initial series was badly formed. This example is a case of an improvement in intellectual status leading to a deterioration in recall. (Several studies cited by Liben, 1977, actually obtain this kind of result.) The main point of Piaget’s theory is not that memory necessarily improves over time—it seldom does— but that memory is influenced by developing intellectual operations, and not just by real events.

Summary

Piaget distinguishes between two types of memory. *Memory in the wider sense* refers to the individual’s ability to retain over time the potential to exhibit learned schemes or operations. *Memory in the specific sense* refers to the individual’s ability to retain over time information concerning particular events, things, or persons. Specific memory may take one of several forms, the most important of which are *recognition* (an impression of familiarity on an encounter with a previously experienced object) and *recall* (evocation of the past through mental representations). Piaget’s general hypothesis is that specific memory is influenced by intelligence—the intellectual operations. Intelligence serves to organize and shape memory. Piaget rejects the classic view in which events are seen to impress themselves on a passive observer, leaving a trace or a simple copy of the reality.

Piaget’s experiments on memory for a series demonstrate that after one week, recall is influenced by the individual’s stage of intellectual development. Presented with a well-formed series, some children recall not what they have seen, but what they know about the series. It is important to note, however, that there is some variability in these results. According to Piaget, there is a general developmental progression from recognition memory to recall. Infants show signs of recognition; recall does not seem to appear until the onset of the semiotic function at about 18 months.

After it appears, recall is influenced by the development of intellectual structures. The general hypothesis states that as intellectual structures develop, they exert corresponding developmental effects on recall. Indeed, under certain circumstances, recall may actually improve over time. Piaget has shown that in the case of seriation, recall becomes more accurate as children advance from one intellectual stage to the next. It is important to note, however, that this result is not easily replicated and that Piaget's theory does not always predict improvement in recall over time. Instead, the main point of Piaget's theory is that memory is influenced and organized (but not necessarily *improved*) by developing intellectual operations, and not simply by real events. Memory is the result of an interaction been knower and known.

CONSCIOUSNESS

We have seen how the child develops operative and figurative aspects of thought. By the age of 7 or 8 years, he achieves some success at classifying and ordering objects, at producing mental images, and at remembering. These cognitive processes, both figurative and operative, work mainly on an unconscious level. Now we will assume a different level of analysis to consider a new topic which Piaget has recently studied, namely, the child's awareness and verbalization of his own thought processes.

In studying the issue of consciousness, Piaget's general strategy is first to have the child solve a problem and second to determine his awareness of the methods of solution (*The Grasp of Consciousness, GC,* 1976b). In one investigation, Piaget used standard seriation tasks, involving such materials as a set of cards varying in height and width, or a set of barrels varying in both height and diameter. Each child's task was to arrange the objects in order of increasing (or decreasing) size. He was told, for example, to "make a nice line of barrels." As soon as the child began to do this, the investigator asked him to describe what he was doing or was about to do. Sometimes the child was asked "how he would explain to a friend what should be done" *(GC,* p. 30I). After the child completed the first series (successful or not), he was asked to repeat it and to describe and explain his actions as he went along. The purpose of this repetition was to ensure that the child knew what was expected of him.

Suppose that a child succeeds at the seriation tasks just described: he produces an accurate ordering in terms of length of the rods or size of the barrels. Given this, we may inquire into the child's *consciousness* or *cognizance* of seriation. It is important to begin by clarifying what is meant by Piaget's usage of *consciousness* or *cognizance.* Piaget uses these terms to refer to the child's ability to produce a coherent verbal account of the mental processes underlying his behavior. By this definition, the child is conscious or cognizant of his thought processes if he says, for example, "I always look for the biggest one, then I put it aside and look for the biggest one out of all the ones that are left." In Piaget's usage, *consciousness* refers to an awareness and verbalization of one's own thought processes. Not only is the conscious child able to do something; he is also explicitly aware of how he does it.[9] Note that Piaget does not use consciousness to refer to the elementary and fleeting perception of the immediate situation. Thus the term is not used to refer to the child's awareness that there are toy barrels on the table or that his hand is moving toward them, and so on. While such elementary awareness appears very early in life and is no doubt highly prevalent, it is not the subject of Piaget's investigation. In brief, Piaget is interested in the child's explicit knowledge of his thought processes, and not merely in the crude awareness of ongoing activities.

Several questions then arise with respect to consciousness. It is especially interesting to inquire into the temporal relations between action and cognizance. There are of course several possibilities. One alternative is that action and cognizance emerge simultaneously. As one develops so does the other, and it is impossible to determine the direction, or even existence, of causality. A second possibility is that consciousness comes first, and thus directs the subsequent action. Perhaps the child first conceptualizes his action and this helps him to perform it. A third possibility is just the reverse. Perhaps successful behavior precedes cognizance of it. The child may be able first to perform certain actions, and only later, upon reflection, does he become aware of his behavior.

The behavior of one of Piagets' subjects, STO, at 6-1, working at seriation, sheds some light on these issues. On his first attempt, STO failed to complete a successful series. He could not arrange cards in order of size and put the smallest ones in the center of the line. He said, "I've made a staircase that goes up or down." The examiner responded that the staircase should go down all the time, "but first tell me how are you going to make it?" STO responded: "I'm going to put the big one, another big one, another big one, the middle-size one, the smaller middle-size one, the smaller middle-size one, and the smaller middle-size one" (*GC*, p. 312). STO proceeded to produce a good series, with only one mistake, which he easily corrected. On subsequent trials, the same sort of thing happened: STO produced good series but poor verbal descriptions.

According to Piaget, this example shows that STO's seriation was far in advance of his consciousness of it. STO could order the cards in a fairly systematic way and yet could refer only in an imprecise manner to "another big one, another big one," or to "the smaller middle-size one, and the smaller middle-size one." Other children exhibit similar behavior. For example, they use an extremely systematic procedure for seriation (like selecting the smallest and then the smallest of all those left) and yet can say only that they first took a small one, then another small one, and so on. Piaget concludes from data like these that, in general, the child's successful activities—including operative activities like seriation—precede cognizance of them. The child can act and think effectively before he can verbalize or be conscious of his actions or thoughts.

How does consciousness of problem solving develop? Piaget proposes that at first the child is only dimly aware of goals. For example, he wants to make a "staircase." The child then gradually develops various strategies for achieving his goal, for example, random placement or systematic selection of the largest. At first, he is quite unaware of these strategies, just as the 3-month-old baby is not conscious of the procedures which he uses for getting his thumb into his mouth. He acts, successfully or unsuccessfully, but does not explicitly analyze his actions. With development, however, the child observes his own activities and reflects on them. He interprets his actions; he tries to "reconstruct" them on the plane of thought. At first, this process of interpretation may lead to distortion and misunderstanding. Piaget has observed many cases in which the initial consciousness was in error—where the child did not accurately see what in fact he had done. But gradually, the reconstruction becomes more and more accurate. The child's reflection on his own activities allows the development of explicit knowledge concerning both his problem-solving processes and the objects under consideration. In this way, the child learns about himself and about the objects surrounding him. He develops abstract concepts that can be verbalized.

Piaget's position has much to recommend it. It seems useful to make a distinction between at least two levels of knowledge. There does seem to be a kind of "action knowledge" or "how-to knowledge" in which we solve problems using means of which we are unaware. Thus STO could seriate, but without consciousness of his method. At the same time, there is also another level of abstract knowledge, in which we can explicitly formulate our methods of solution and even the principles underlying them. Thus a child cannot only seriate but explicitly understands the principles which he uses. The process of transforming action knowledge into abstract knowledge may be crucial for human learning. There is a good deal of wisdom built into our behavior, and a major task for learning may consist in making explicit what in a sense we already know unconsciously.

While these are useful points, Piaget's investigations in this area seem to suffer from a major weakness, namely, an overreliance on verbalizations as a source of evidence. In these studies, verbalization is taken as the main, or even only, source of evidence for consciousness or cognizance. Thus STO is said to lack consciousness of his actions, since his language is inadequate. But STO's repetitive use of vague terms like "the smaller middle-size one" may not accurately reflect the true level of his consciousness. Seriation is hard to express in words, and perhaps STO could conceptualize it but was unable to offer adequate descriptions of the process. Piaget's interpretation seems weak in this regard. At the same time, despite the difficulties, Piaget's research raises extremely provocative issues requiring a good deal of further study.

GENERAL CONCLUSIONS

While criticisms may and should be made, and while revisions are necessary, Piaget's theory is an enormously significant accomplishment. Indeed, on reviewing Piaget's later work on the child from 2 to 11, one is struck above all by the incredible creativity and diversity of his contribution. Between 1940 and 1980, Piaget revolutionized the study of the child. He introduced a score of fascinating problems and experimental tasks— conservation is only one example—which for a long time dominated research in child psychology. More important, he offered an extraordinarily deep and subtle theory of cognitive development, which continues to inform our understanding of the mind's growth.

Notes

1 See H. P. Ginsburg, "The Clinical Interview in Psychological Research: Aims, Rationales, Techniques," *For the Learning of Mathematics,* Vol. 3 (1981), pp. 4-11, and S. Opper, "Piaget's Clinical Method," *Journal of Children's Mathematical Behavior,* Vol. 1 (1977), pp. 90-107.

2 See, for example, R. Gelman and C. R. Gallistel, *The Young Child's Understanding of Number* (Cambridge, Mass.: Harvard University, Press, 1978), Chap. 3.

3 Our exposition of Grouping I is simplified and incomplete: for example, we have defined only one binary operator. We have kept the mathematical development at a very informal level. The reader interested in pursuing the matter should see Jean Piaget, *Traite de Logique* (Paris: Colin, 1949), and alsoj. B. Grize's formalization of Piaget's system as described in E. W. Beth and Jean Piaget, *Mathematical Epistemology and Psychology* (Dordrecht, Holland: D. Reidel Publishing Company, 1966).

4 Although it does not seem to help with conservation, counting is far from useless in children's arithmetic. Hebbeler has shown, for example, that young children make very good use of counting in doing addition. See K. Hebbeler, "Young Children's Addition," *Journal of Children's Mathematical Behavior*, Vol. 1 (1977), pp. 108-21.

5 Strictly speaking, in the case of number Piaget uses a somewhat different logico-mathematical model, called the *Group*. The essential difference between the Groupings and the Group is that the fifth Grouping operation, tautology (e.g., A + A = A), is not used in the Group. Tautology does not apply to number since there A + A = 2A, not A. Therefore, the Group must be used for number.

6 Recently, Gelman and Baillargeon (1983, p. 171) have argued that the phenomenon of invariant sequence is not as clear-cut as Piaget suggests. They describe research showing that in some areas some children do not exhibit the stages in the order predicted by Piaget. This seems to present serious difficulties for the theory.

7 There can be instances of false recall. Piaget himself falsely remembered being the object of an abortive kidnap attempt when he was a child.

8 Piaget's exposition of the classic view probably refers to theorists like Ebbinghaus, who in the nineteenth century invented nonsense syllables and spent many years of his life memorizing them himself. He was his only subject and deserves some sort of prize for an immense capacity for boredom. In recent years, however, theorists of memory have given up both the inclination themselves to memorize nonsense syllables (although may require their subjects to do it) and theoretical accounts which treat the subject as passive. Many modern theories are in substantial agreement with Piaget on the issue of activity. For a comparison of Piaget's theory with others, as well as an excellent critique of Piaget's work, see L. Liben, "Memory from a Cognitive-Developmental Perspective: A Theoretical and Empirical Review," in *Knowledge and Development*, W. F. Overton and J. M. Gallagher, eds. (New York: Plenum Press, 1977), Vol. I, pp. 14-9-203.

9 Recently, Flavell and others have been investigating a similar topic, which they term "meta cognition," and which involves the child's knowledge about his own knowledge. (For a review, see J. H. Flavell, *Cognitive Development* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1985.) An example is whether the child is aware of using systematic strategies to aid in memory.